StruSoft FEM-Design

FEM-Design

Dynamic Response of Moving Load
Theory and Verifications

version 1.0
2019

StruSoft AB
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Dynamic Response of Moving Load
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Contents

List of symbols

$\boldsymbol{C}$	damping matrix
f_i	i-th eigenfrequency
$\boldsymbol{K}$	stiffness matrix
L_p	path length
$\boldsymbol{M}(t)$	diagonal (lumped) mass matrix (in function of time)
$\boldsymbol{M}_g$	constant mass matrix based on the converted constant loads
$\boldsymbol{M}_m(t)$	mass matrix of the moving load in function of time
n	division number of the path
$\boldsymbol{q}(t)$	excitation force vector in function of time (moving load)
t	time
T	time period
v	velocity of the moving loads
$\boldsymbol{x}(t)$	displacement vector in function of time
$\dot{\boldsymbol{x}}(t)$	velocity vector in function of time
$\ddot{\boldsymbol{x}}(t)$	acceleration vector in function of time
$x_{i,dyn}$	vertical dynamic translational displacement of the i-th node
$x_{i,stat}$	vertical static translational displacement of the i-th node
$x_{j,stat}$	maximum vertical static translational displacement on the path
α	Rayleigh damping matrix coefficient
β	Rayleigh damping matrix coefficient
Δs	path step
Δt	time step
ξ_i	critical damping ratio of the i-th eigenfrequency
ω_i	i-th angular natural frequency

1 Dynamic effect calculation of moving load

1.1 The purpose of this type of moving load dynamic calculation

Calculation of the dynamic effect of loads (masses) which are moving on bridges, overpasses or anywhere. On a pre-defined path the load (mass) or load groups (masses) are moving along the structure with constant specific velocity. This kind of excitation will result a dynamic response of the structure. The result by this calculation method is primarily the calculation of the dynamic factor, but also the dynamic displacements and accelerations on the structure. See EN 1991-2 about actions on structures as well.

1.2 The solution method of the dynamic differential equation system

The second order linear inhomogeneous differential equation system is the following, see Ref. [1]:

$$\boldsymbol{M}\,\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\,\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\,\boldsymbol{x}(t)=\boldsymbol{q}(t) \quad ,$$

where $\boldsymbol{M}$ is the diagonal mass matrix of the permanent loads, mass points and optionally the mass of the moving load. $\boldsymbol{C}$ is the so-called Rayleigh damping matrix. $\boldsymbol{K}$ is the linear global structural stiffness matrix. $\boldsymbol{q}(t)$ is the load vector calculated at t time based on the specified moving loads. $\boldsymbol{x}(t)$ is the displacement vector at time t. $\dot{\boldsymbol{x}}(t)$ is the velocity vector at time t. $\ddot{\boldsymbol{x}}(t)$ is the acceleration vector at time t.

The solution of the differential equation of motion is achieved by the direct integration method. The used integration rule is the so-called Newmark or Wilson-Θ method depending on the settings and according to Ref. [2].

1.3 The mass matrix

Usually the mass matrix only contains the mass of the structure. It is also possible to consider the mass of the moving load. It is an adjustable settings by the setup of the moving load calculation in FEM-Design. In this case the mass matrix is as follows:

$$\boldsymbol{M}(t)=\boldsymbol{M}_g+\boldsymbol{M}_m(t) \quad ,$$

where $\boldsymbol{M}_g$ is the mass matrix calculated from the weight of the structure (mainly from permanent loads) which is constant during the calculation (time independent). $\boldsymbol{M}_m(t)$ is a mass matrix calculated in each position of the moving load.

NOTE: Mass of the moving load should be considered only if the value of the moving weight is equal to or greater than 10% of the weight of the structure. The time-dependent mass matrix significantly increases the computational time.

1.4 The Rayleigh damping matrix

The so-called Rayleigh damping matrix contains a mass-proportional and a stiffness-proportional part.

$$\boldsymbol{C} = \alpha\,\boldsymbol{M} + \beta\,\boldsymbol{K}$$

If the mass matrix is time-dependent, then the damping matrix also becomes:

$$\boldsymbol{C}(t) = \alpha\left(\boldsymbol{M}_{\boldsymbol{g}} + \boldsymbol{M}_{\boldsymbol{m}}(t)\right) + \beta\,\boldsymbol{K}$$

The first term (mass-proportional part) refers to extrenal damping (as a point support), while the second term (stiffness-proportional part) refers to coupled internal damping (as a point-to-point relationship). For further information see Ref. [1] and Fig. 1.

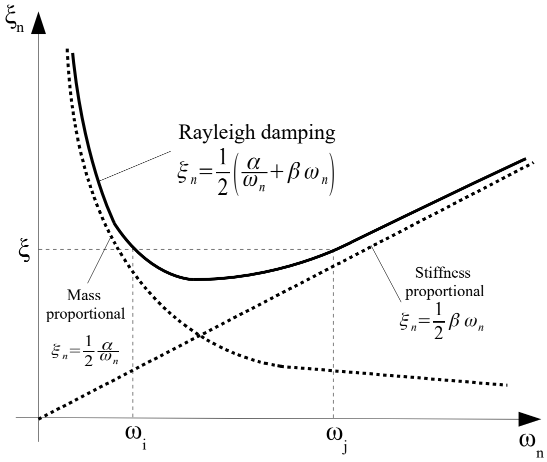

Figure 1 – Interpretation of Rayleigh damping

The necessary α and β parameters depend on the analyzed structure. The recommended values according to Ref. [1-2] are as follows:

$$\xi_i = \frac{1}{2}\left(\frac{\alpha}{\omega_i} + \beta\,\omega_i\right)$$

We can rearrange this equation:

$$\alpha + \beta\,\omega_i^2 = 2\,\omega_i\,\xi_i$$

Based on this equation the α and β parameters can be calculated by specifying 2-2 angular frequencies and damping ratios. In the expression ω_i is the *i*-th angular frequency and ξ_i is the *i*-th damping ratio related to the *i*-th mode shape.

For example:

$$\xi_1 = 0.03 \quad ; \quad \omega_1 = 4\ \text{rad/s}$$

$$\xi_2 = 0.12 \quad ; \quad \omega_2 = 17\ \text{rad/s}$$

Using the former equation to the 2-2 angular frequencies and damping ratios:

$$\alpha + 16\beta = 0.24$$

$$\alpha + 289\beta = 4.08$$

Based on these two equations:

$$\alpha = 0.01498 \quad ; \quad \beta = 0.01405$$

According to Ref. [1] the variations of modal damping ratios with natural frequencies are not consistent with experimental data that indicate roughly the same damping ratios for several vibration modes of a structure. If both modes are assumed to have same damping ratio ξ, which is reasonable based on experimental data, then the parameters can be calculated as follows:

$$\alpha = \xi \frac{2\omega_i \omega_j}{\omega_i + \omega_j} \quad \text{and} \quad \beta = \xi \frac{2}{\omega_i + \omega_j}$$

Or expressed with the natural eigenfrequencies:

$$\alpha = \xi \frac{4\pi f_i f_j}{f_i + f_j} \quad \text{and} \quad \beta = \frac{\xi}{\pi} \frac{1}{f_i + f_j}$$

SUGGESTION: It is advisable to calculate first the eigenfrequencies considering only the mass in the global Z direction and select the two specific eigenfrequencies (and the associated vibration shapes) based on this calculation. The most obvious method is to choose the two eigenfrequencies which provide the highest effective masses. In FEM-Design the effective masses after the eigenfrequency calculation can be found in the setup menu of the seismic calculation.

If someone would like to consider only the so-called equivalent Kelvin-Voigt damping, in that case the Rayleigh damping parameters:

$$\alpha = 0 \quad \text{and} \quad \beta = \frac{\xi}{\pi f_i}$$

1.5 Time step and preliminary conditions

By a discrete model, assuming an evenly spaced load, the time step can be calculated based on the specified velocity and the division number of the moving load on the specified path:

$$\Delta t = \frac{\Delta s}{v}$$

The initial conditions are necessary to solve the differential equation system. In FEM-Design it is considered in the following way. The initial displacement at $t = 0$ time is $x(0) = x_0$. The x_0 is the static displacement of the structure calculated under the moving load at the starting position on the path. The initial velocity is $\dot{x}(0)=v_0=0$. The initial acceleration is $\ddot{x}(0)=a_0=0$.

We can see from the algorithms (Newmark or Wilson-Θ) of the direct solution of the differential equation in Ref. [1] that the controll parameter is the Δt time step. The correctly chosen Δt is essential to obtain sufficiently accurate results and to the stability of the numerical solution.

In FEM-design the input data to the moving mass dynamic response calculation is the division number of the moving load position on the specified path. Therefore based on the specified velocities this division number of the moving load on the path controls the time step size of the numerical algorithm with the Δs distance between the moving load adjacent positions (see the previous equation).

We recommend to calculate the Δt using the vibration time period (T) from the maximum effective mass mode shape with the following method.

Let L_p be the length of the path and estimate the time step using the next formula:

$$\Delta t=\frac{T}{20}=\frac{\Delta s}{v}$$

Thus the path division number (n) should be minimum, see Fig. 2:

$$n=\frac{20\,L_p}{v\,T}$$

1.6 Finite element calculation model

Dynamic calculations are not sensitive to the “fine” or “standard” finite element type settings. There are no internal force peaks here, thus we recommend to use “standard” element group to speed up the calculation time. It may be worth to compare the values of the eigenfrequencies from the same model based on the two types of finite element group.

1.7 The path of the moving load

More paths can be calculated individually but it means that the results from different paths cannot be combined together.

Only the 'division number' can be considered. That is, the step length on each path must be constant. Based on this the time step (Δt) can be calculated. The load is not reversed because it makes no sense due to the constant velocity vector of the moving mass (see. Fig. 2).

It is advisable during the specification of the path to ensure that the load is just touching the structure in the starting position and touching or leaving the final necessary position.

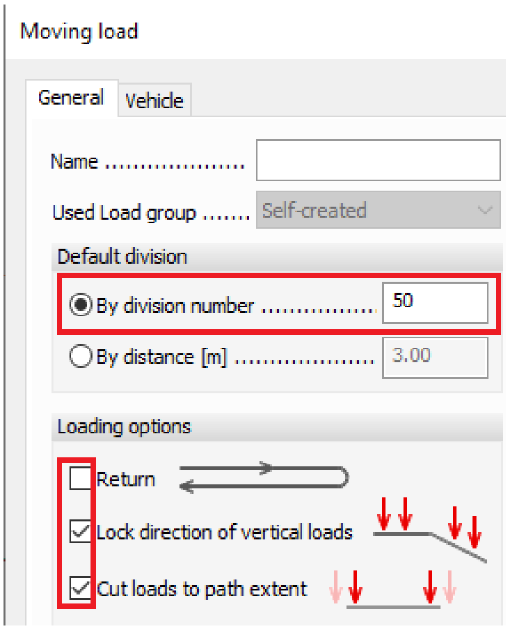

Figure 2 – The setting of the applied moving load

1.8 Calculations and results

The analysis setup dialog can be seen in Fig. 3. You can set here the following options:

- Method of the integration scheme. Newmark or Wilson-Θ.
- Decision about the moving load mass conversion during the calculation.
- Minimum and maximum value of the analyzed velocity range. The number of velocity division gives the considered discrete values of the velocities in the given range.
- The Rayleigh damping coefficients based on the mentioned recommendation in Chapter 1.4 can be given here.
- If the user defined several moving loads in the model then here can be selected which moving loads will be involved into the dynamic response calculation.

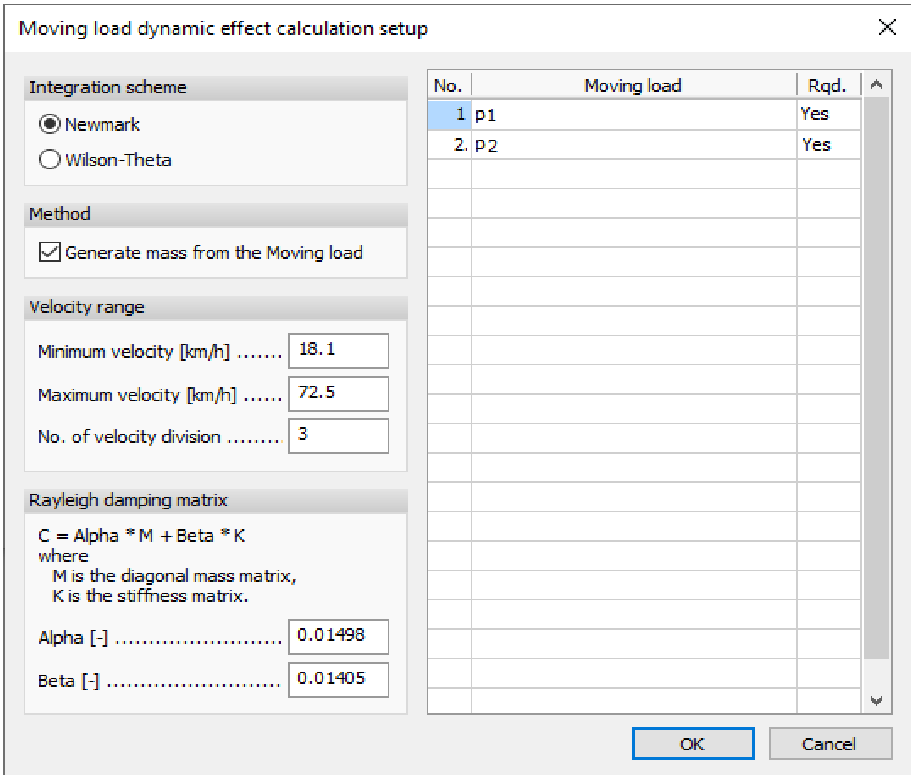

Figure 3 – The moving load setup

After the dynamic response calculation of the moving loads the following results will be available:

- Static displacements with graph or color palette view by the different load positions of the moving load and the maximum value. Detailed graphical function results are also available to a single node.
- Dynamic displacements with graph or color palette view by the different load positions of the moving load and the maximum value. Detailed graphical function results are also available to a single node.
- Accelerations color palette view by the different load positions of the moving load and the maximum value. Detailed graphical function results are also available to a single node.
- Dynamic factor color palette view by the different load positions of the moving load and the maximum value. Detailed graphical function results are also available to a single node.
- Normalised dynamic factor color palette view by the different load positions of the moving load and the maximum value. Detailed graphical function results are also available to a single node.

NOTE: As you can see two kinds of dynamic factor are available by the results.

The regular dynamic magnification factor (so-called dynamic factor) at a given *i*-th node is calculated as the ratio of the dynamic and static displacements caused by the moving load along the path.

$$\frac{x_{i,dyn}}{x_{i,stat}}$$

The normalised dynamic factor will be calculated in the same way, but in the denominator we consider the maximum static displacement of the structure instead of the actual static displacement of the specific node.

$$\frac{x_{i,dyn}}{x_{j,stat,max}}$$

1.9 Restrictions of the calculation

Since this type of dynamic calculation is linear, all non-linearity effects will be neglected during this calculation e.g.:

- Uplift
- Cracked section analysis
- Plastic calculation
- Construction stage calculation
- Diaphragm
- Non-linear soil
- 2nd order effect

2 Verification examples

2.1 Dynamic response of a moving mass point on a simply supported beam

Example is taken from Ref. [3]. Let's take a simply supported beam with one moving mass point on it, see. Fig. 4.

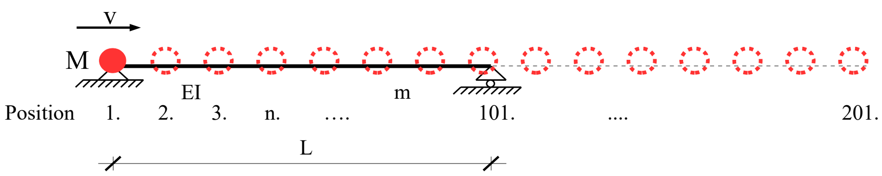

Figure 4 – The geometry and the moving mass positions on the path during the calculation

Inputs (as you can see in this case the moving object will be a "force" but its mass will be considered during the dynamic calculation):

Analyzed position	Mid-span of the beam
The distributed mass of the beam	m = 312 kg/m
Mass of the moving mass	M = 1248 kg
Elastic modulus of the beam	E= 206 GPa
Span length	L = 20 m
Analyzed constant velocities of the moving mass	v_1 = 18.12 km/h
	v_2 = 36.24 km/h
	v_3 = 72.48 km/h
Critical damping ratio	ξ = 0 %
Path length	L_p = 40 m
Cross-section	200 mm x 200 mm square

In FEM-Design by this example during the direct integration calculation the damping was neglected (thus in the Rayleigh damping matrix the α = 0 and β = 0), therefore the system is undamped. In the FEM model we split the beam into 50 parts to consider a relevant continuous mass distribution, and 201 mass point positions, see Fig. 4. The results belong to those positions which are not on the structure will give us the free vibration response of the structure, see Fig. 7.

The first three eigenfrequencies of a simply supported beam with continuous distributed mass.

$$f_1=\frac{\pi}{2\cdot L^2}\sqrt{\frac{EI}{m}}=\frac{\pi}{2\cdot 20^2}\sqrt{\frac{206000000000\cdot 0.2^4/12}{312}}=1.165\,\mathrm{Hz}$$

$$f_2=\frac{\pi}{2\cdot\left(\frac{L}{2}\right)^2}\sqrt{\frac{EI}{m}}=\frac{\pi}{2\cdot\left(\frac{20}{2}\right)^2}\sqrt{\frac{206000000000\cdot 0.2^4/12}{312}}=4.661\,\mathrm{Hz}$$

$$f_3 = \frac{\pi}{2 \cdot \left(\frac{L}{3}\right)^2} \sqrt{\frac{EI}{m}} = \frac{\pi}{2 \cdot \left(\frac{20}{3}\right)^2} \sqrt{\frac{206000000000 \cdot 0.2^4 / 12}{312}} = 10.49\,\text{Hz}$$

According to FEM-Design calculation the first three eigenfrequencies of the beam are as follows, see Fig. 5:

$f_{FEM\,1} = 1.165\,\text{Hz}$; $f_{FEM\,2} = 4.658\,\text{Hz}$ $f_{FEM\,3} = 10.47\,\text{Hz}$

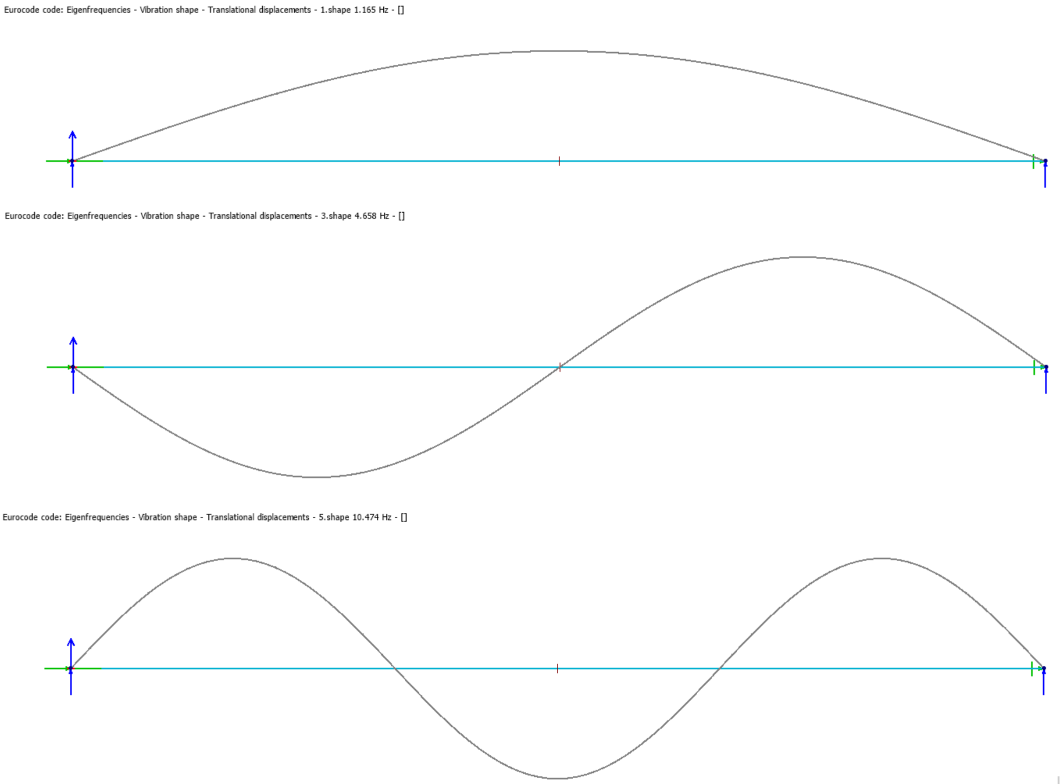

Figure 5 – The first three eigenshapes in FEM-Design

The critical velocity:

$$v_{cr} = 2\,L\,f_1 = 2 \cdot 20 \cdot 1.165 = 46.6\,\frac{\text{m}}{\text{s}} = 167.76\,\frac{\text{km}}{\text{h}}$$

The analytical solutions based on Ref. [3] are the normalised dynamic factors of the beam mid-span under different velocities. These values are very close to the FEM-Design numerical calculations.

The maximum static deflection of the mid-span if the mass is at the mid-span:

$$e_{max} = \frac{F\,L^3}{48\,EI} = \frac{M\,g\,L^3}{48\,EI} = \frac{1248 \cdot 9.81 \cdot 20^3}{48 \cdot 206000000000 \cdot 0.2^4 / 12} = 0.07429\,\text{m}$$

The maximum static deflection at mid-span based on FEM-Design calculation (see also Fig. 7):

$$e_{max}^{FEM} = 0.07431\,\mathrm{m}$$

The maximum normalized dynamic factor at the mid-span under v_1 velocity according to Ref. [3]:

$$\left(\frac{Z_1^{\mathrm{dynamic}}}{Z_{\mathrm{max}}^{\mathrm{static}}}\right)_{\mathrm{max}} = 1.1$$

This value based on FEM-Design, see Fig. 6-7:

$$\left(\frac{Z_1^{\mathrm{FEMdynamic}}}{Z_{\mathrm{max}}^{\mathrm{FEMstatic}}}\right)_{\mathrm{max}} = 1.096$$

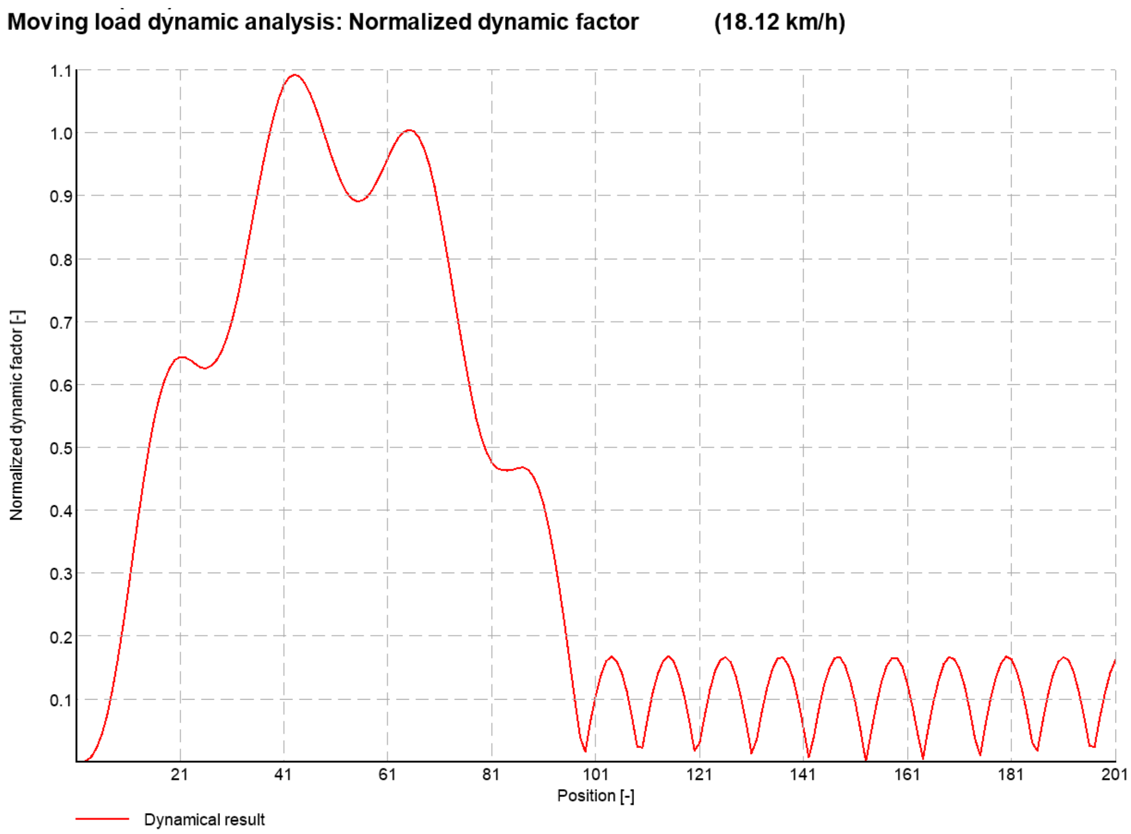

Figure 6 – The normalized dynamic factor diagram based on FEM-Design with v_1 velocity

Fig. 7 shows the static and dynamic displacements at the mid-span under the different load positions with v_1 velocity.

In Fig. 7 we can see the static and dynamic maximum deflection at the mid-span.

$$e_{dynamic\,max}^{FEM\;v_1} = 0.08139\,\mathrm{m}$$

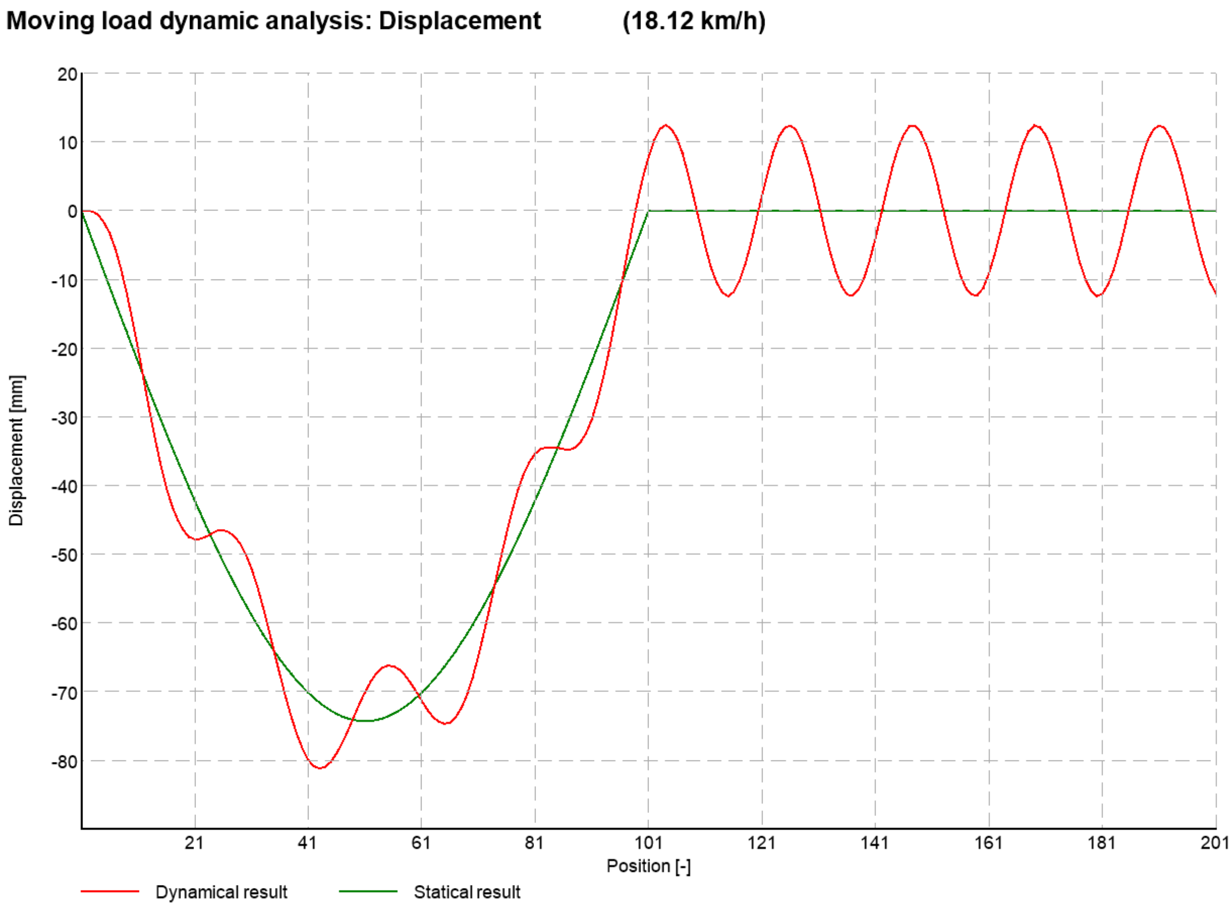

Figure 7 – The dynamic and static displacements of the mid-span in function of mass positions with v_1 velocity with FEM-Design

The maximum normalized dynamic factor at the mid-span under v_2 velocity:

$$\left(\frac{Z_2^{\text{dynamic}}}{Z_{\text{max}}^{\text{static}}}\right)_{\text{max}} = 1.21$$

This value based on FEM-Design, see Fig. 8:

$$\left(\frac{Z_2^{\text{FEMdynamic}}}{Z_{\text{max}}^{\text{FEMstatic}}}\right)_{\text{max}} = 1.21$$

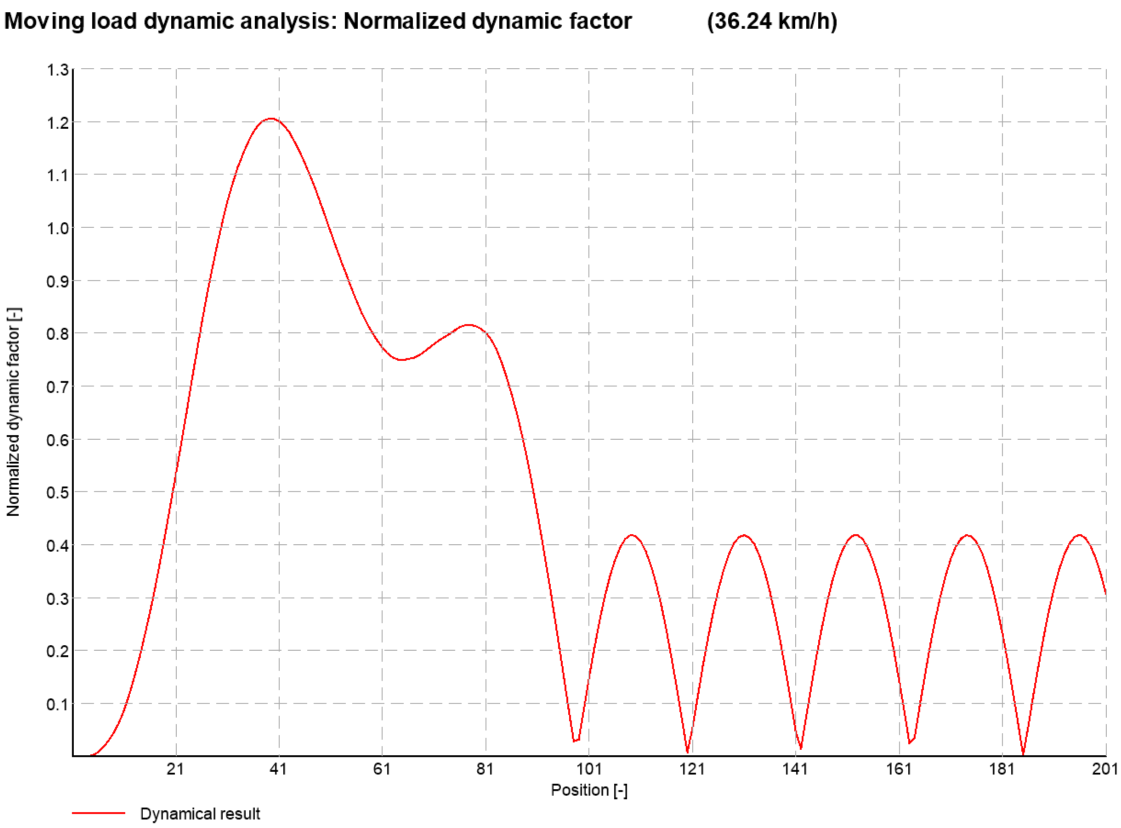

Figure 8 – The normalized dynamic factor diagram based on FEM-Design with v_2 velocity

Fig. 9 shows the static and dynamic displacements at the mid-span under the different load positions with v_2 velocity.

About Fig. 9 we can see the static and dynamic maximum deflection at the mid-span.

$$e_{dynamic\ max}^{FEM\ v_2} = 0.08991\ \text{m}$$

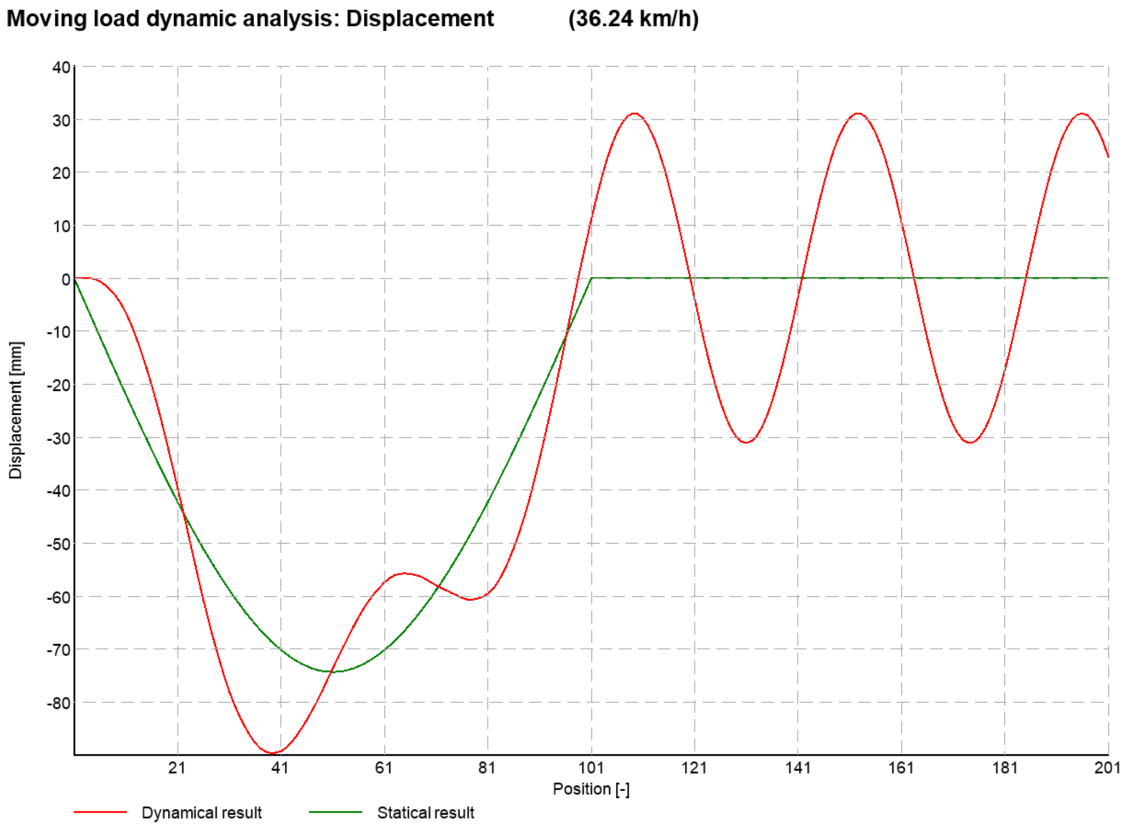

Figure 9 – The dynamic and static displacements of the mid-span in function of mass positions with v_2 velocity with FEM-Design

The maximum normalized dynamic factor at the mid-span under v_3 velocity:

$$\left(\frac{Z_3^{\text{dynamic}}}{Z_{\text{max}}^{\text{static}}}\right)_{\text{max}} = 1.85$$

This value based on FEM-Design, see Fig. 10:

$$\left(\frac{Z_3^{\text{FEMdynamic}}}{Z_{\text{max}}^{\text{FEMstatic}}}\right)_{\text{max}} = 1.76$$

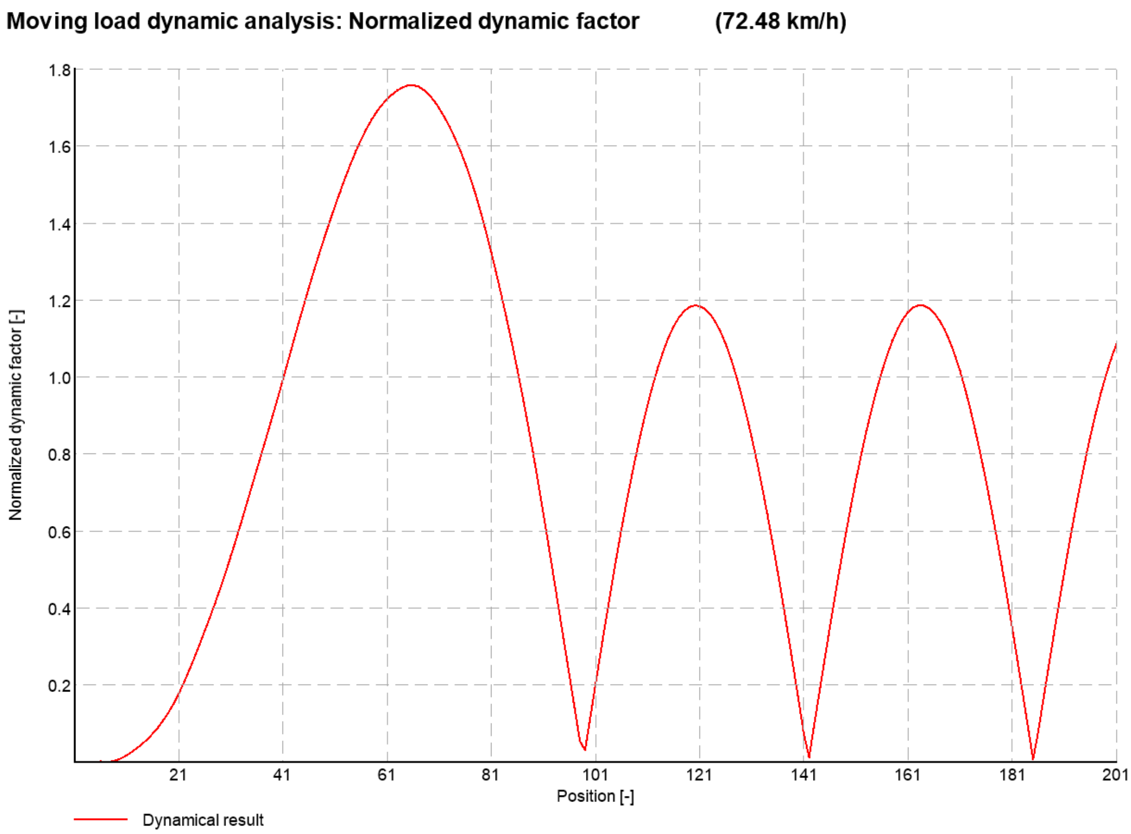

Figure 10 – The normalized dynamic factor diagram based on FEM-Design with v_3 velocity

Fig. 11 shows the static and dynamic displacements at the mid-span under the different load positions with v_3 velocity.

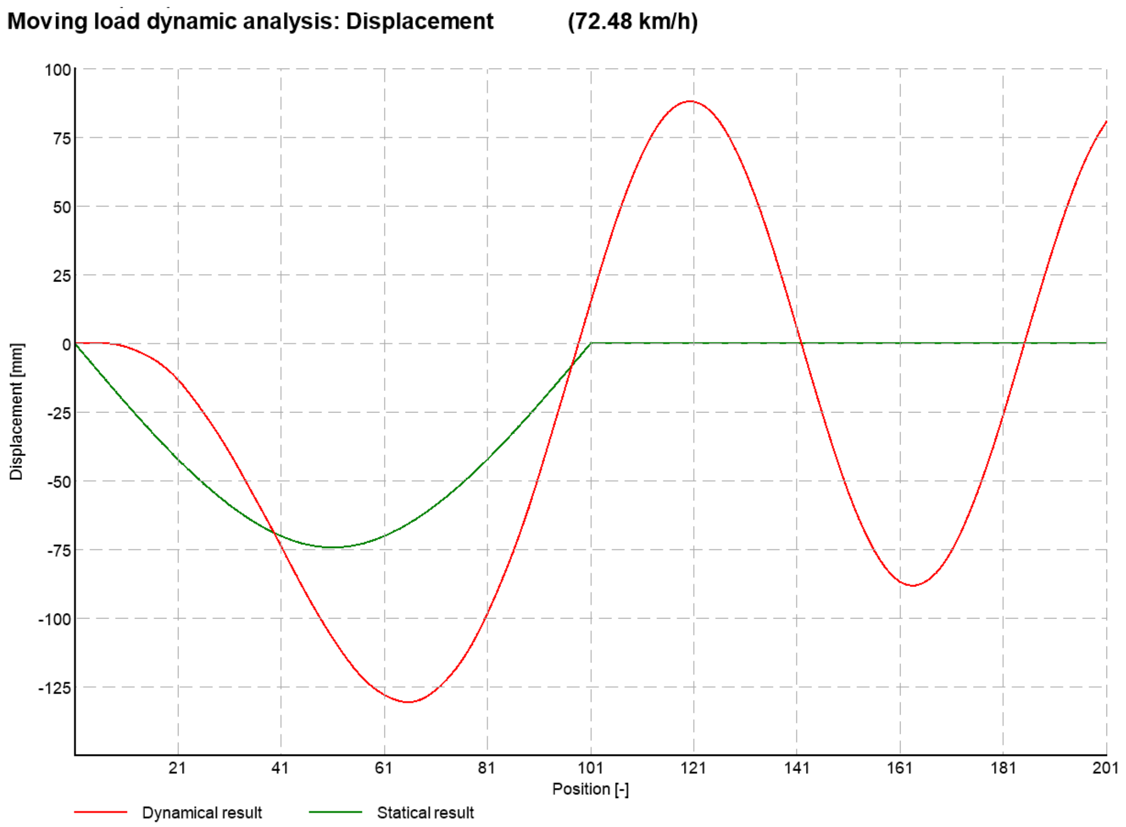

Figure 11 – The dynamic and static displacements of the mid-span in function of mass positions with v_3 velocity with FEM-Design

About Fig. 11 we can see the static and dynamic maximum deflection at the mid-span.

$$e_{dynamic\,max}^{FEM\ v_3} = 0.1310\,\text{m}$$

We can say that there are good agreements between the results. On the FEM-Design dynamic deflection results we can see the free vibration behaviour of the structure as well.

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst190x/models/8.5 Dynamic response of a moving mass point on a simply supported beam.str

2.2 Dynamic response of a moving single force on a simply supported beam

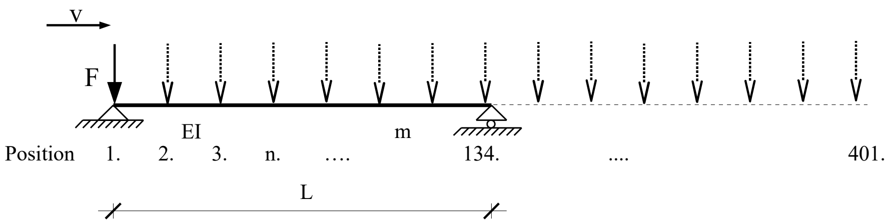

Figure 12 – The geometry and the moving load positions on the path during the calculation

Inputs:

Analyzed position	Mid-span of the beam
The distributed mass of the beam	μ = 7950 kg/m
The concentrated moving force	F = 480 kN
Bending stiffness	EI = $4.26 \cdot 10^7$ kNm^2
Span length	L = 30 m
Analyzed constant velocities of the moving force	v_1 = 50 m/s = 180 km/h
	v_2 = 100 m/s = 360 km/h
Damping	ξ = 0 %
Path length	L_p = 90 m

In this calculation we neglect the mass effect of the moving force, because based on Ref. [4] we can get the analytical solution of the problem directly. Fig. 12 shows the problem with the considered moving force, positions and geometry.

The first eigen and angular natural frequency based on closed form with Bernoulli beam theory:

$$f_1 = \frac{\pi}{2 \cdot L^2}\sqrt{\frac{EI}{m}} = \frac{\pi}{2 \cdot 30^2}\sqrt{\frac{4.26 \cdot 10^7}{7.95}} = 4.040\ \text{Hz} \quad ; \quad \omega_{01} = 2\pi f_1 = 25.38\frac{\text{rad}}{\text{s}}$$

The first eigen and angular natural frequency based on FEM-Design calculation with Timoshenko beam theory:

$$f_{FEM\ 1} = 3.997\ \text{Hz} \quad ; \quad \omega_{FEM\ 01} = 2\pi f_1 = 25.11\frac{\text{rad}}{\text{s}}$$

The analytical solution of the deflection of the beam in function of time and load position according to Ref. [4] if the moving force is on the structure can be calculated with the following formula:

$$e(x,t) = \sum_{r=1}^{\infty} v_r(x)\eta_r(t)$$

where:

$$v_r(x)=\sqrt{\frac{2}{\mu L}}\sin\frac{r\pi x}{L} \quad ; \quad \eta_r(t)=\frac{F}{\omega_{0r}}\sqrt{\frac{2}{\mu L}}\frac{1}{\left(\frac{r\pi v}{L}\right)^2-\omega_{0r}^2}\left(\frac{r\pi v}{L}\sin\omega_{0r}t-\omega_{0r}\sin\frac{r\pi v}{L}t\right)$$

In addition to the above mentioned parameters, *r* is the number of the eigenfrequencies.

At the midspan only the odd number of eigenfrequencies have effect on the final solution. In this case the dominant eigenfrequency is the first one, thus at the analytical solution we will use only the first eigenfrequency by the summation. In the complete analytical solution all of the eigenfrequencies have effect on the final solution, but in this case the rest of them have neglectable influence.

If $0 < t < L/v$ the approximate deflection at the mid-span in function of time:

$$e\left(\frac{L}{2},t\right)=\frac{2}{\mu L}\frac{F}{\omega_{01}}\frac{1}{\left(\frac{1\pi v}{L}\right)^2-\omega_{01}^2}\left(\frac{1\pi v}{L}\sin\omega_{01}t-\omega_{01}\sin\frac{1\pi v}{L}t\right)$$

The derivative of this function (velocity):

$$\dot{e}\left(\frac{L}{2},t\right)=\frac{2}{\mu L}\frac{F}{\omega_{01}}\frac{1}{\left(\frac{1\pi v}{L}\right)^2-\omega_{01}^2}\frac{\omega_{01}\pi v}{L}\left(\cos\omega_{01}t-\cos\frac{\pi v}{L}t\right)$$

In that time value when the force is leaving the structure the initial values of the remaining free vibration part of the solution:

In case of v_1:

$$t_1=\frac{L}{v_1}=\frac{30}{50}=0.6\,\mathrm{s}$$

Displacement:

$$e_{\mathrm{init1}}=e\left(\frac{L}{2},0.6\right)=\frac{2}{7950\cdot 30}\frac{480000}{25.38}\frac{1}{\left(\frac{1\pi 50}{30}\right)^2-25.38^2}\left(\frac{1\pi 50}{30}\sin 25.38\cdot 0.6-25.38\sin\frac{1\pi 50}{30}\cdot 0.6\right)=$$
$$=-0.0006217\,\mathrm{m}$$

Velocity:

$$\dot{e}_{\mathrm{init1}}=\dot{e}\left(\frac{L}{2},0.6\right)=\frac{2}{7950\cdot 30}\frac{480000}{25.38}\frac{1}{\left(\frac{1\pi 50}{30}\right)^2-25.38^2}\frac{25.38\cdot\pi 50}{30}\left(\cos 25.38\cdot 0.6-\cos\frac{\pi 50}{30}0.6\right)$$
$$=-0.003861\,\mathrm{m/s}$$

In case of v_2:

$$t_2 = \frac{L}{v_2} = \frac{30}{100} = 0.3\,\text{s}$$

Displacement:

$$e_{\text{init2}} = e\left(\frac{L}{2}, 0.3\right) = -0.0030183\,\text{m}$$

Velocity:

$$\dot{e}_{\text{init2}} = \dot{e}\left(\frac{L}{2}, 0.3\right) = -0.097608\,\text{m/s}$$

After the moving force has just reached the end of the beam the dynamic behaviour of the beam will be a free vibration (if no damping is considered). With the mentioned initial values based on that time value when the force will be at the end of the beam the function of the free vibration can be given:

If $L/v < t$ the approximate deflection at the mid-span in function of time:

$$e\left(\frac{L}{2}, t\right) = e_{\text{init}} \cos\left[\omega_{01}\left(t - \frac{L}{v}\right)\right] + \frac{\dot{e}_{\text{init}}}{\omega_{01}} \sin\left[\omega_{01}\left(t - \frac{L}{v}\right)\right]$$

Fig. 13 shows the deflection function of the given analytical solution with the two different velocities.

Fig. 14-15 shows the dynamic and static deflection of the mid-span under the moving load based on FEM-Design calculation with the two different velocities.

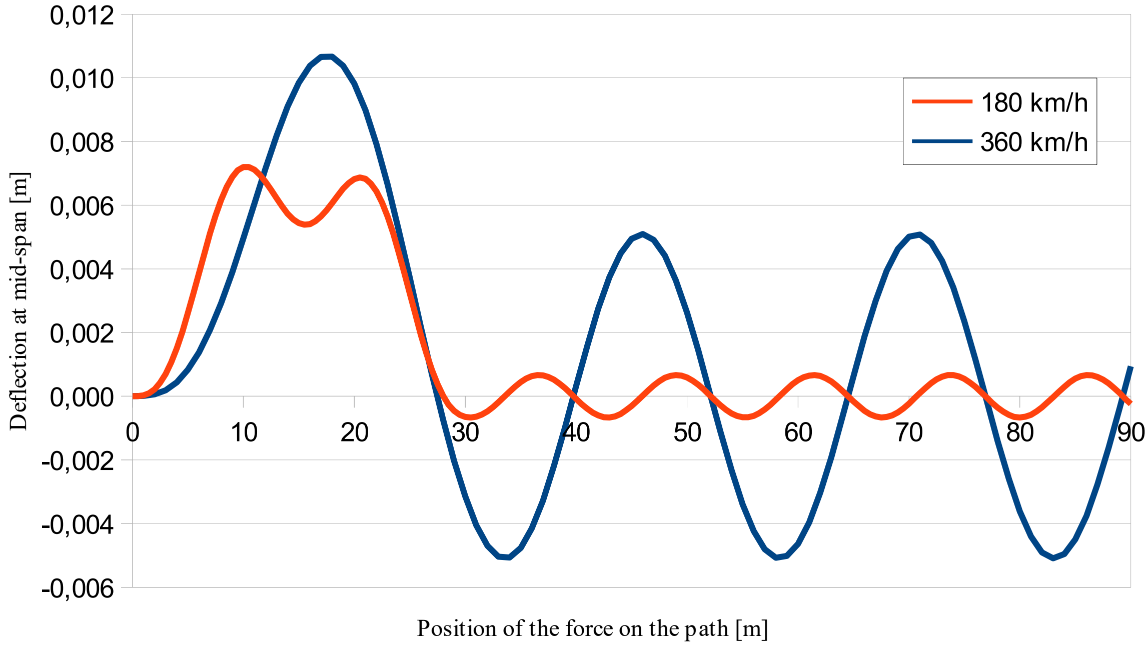

Figure 13 – The dynamic deflection of the mid-span with the two given velocities according to the analytical solution

We can see in Fig. 13-15 that after the position when the load has just left the beam the dynamic response is free vibration.

According to the given solutions we can say that the FEM-Design result are identical with the analytical solution.

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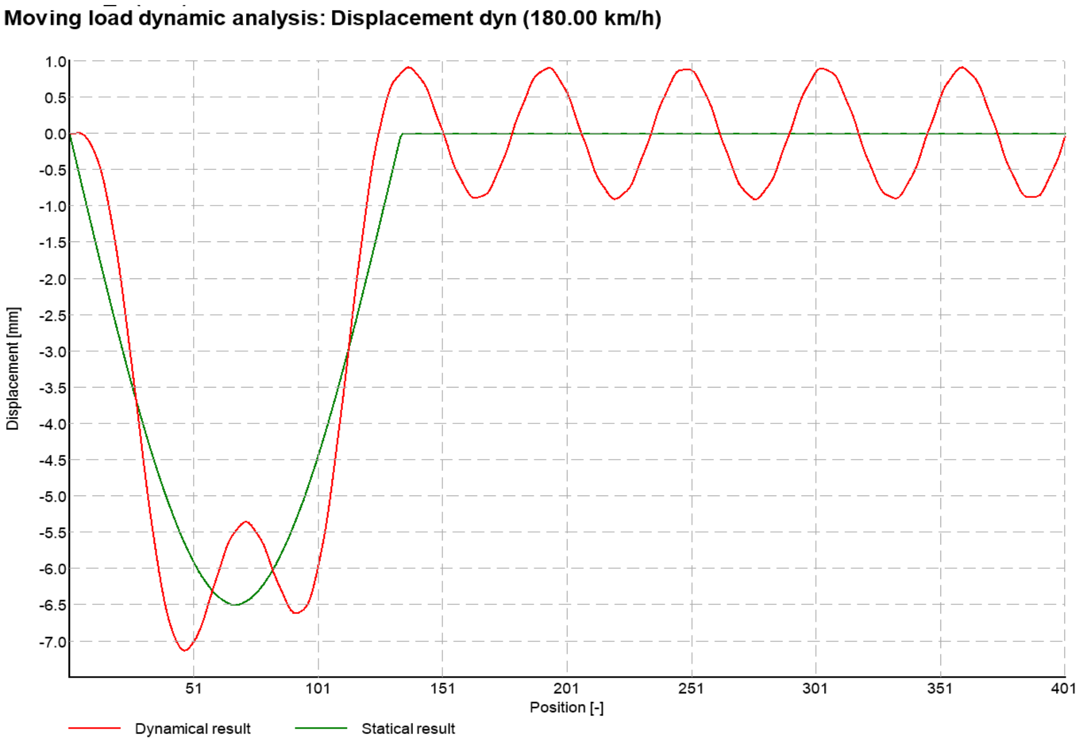

Figure 14 – The dynamic and static displacements of the mid-span in function of force positions with v_1 velocity with FEM-Design

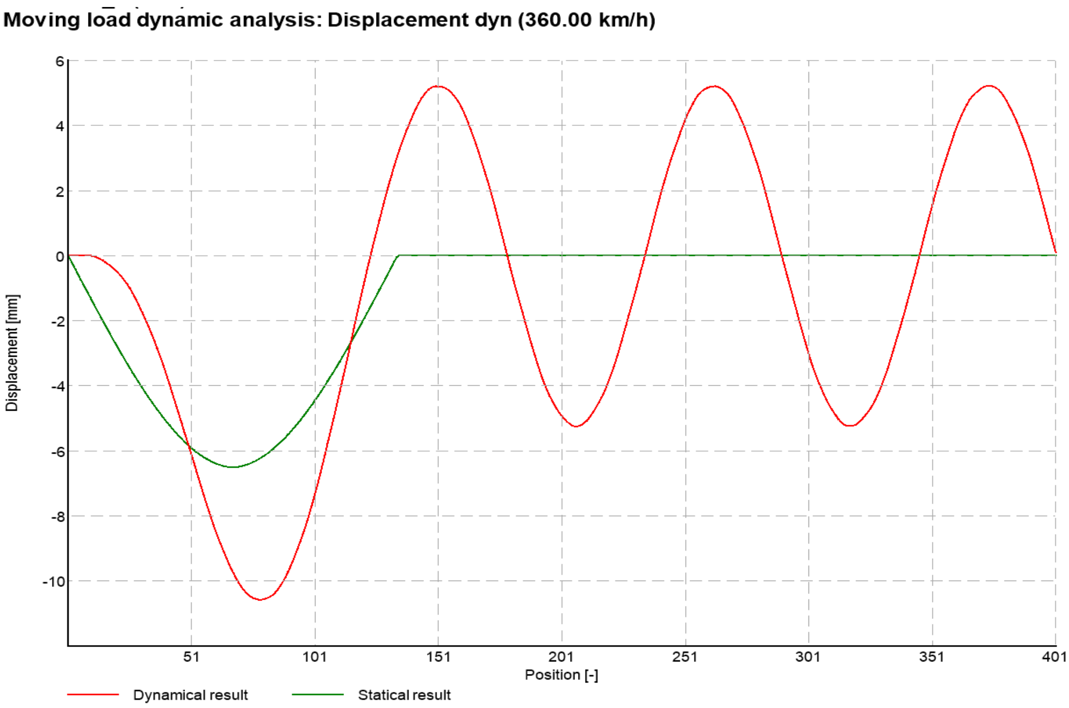

Figure 15 – The dynamic and static displacements of the mid-span in function of force positions with v_2 velocity with FEM-Design

2.3 Dynamic response of a moving force group on a simply supported beam

Inputs:

Analyzed position	Mid-span of the beam
The distributed mass of the beam	m = 7950 kg/m
Bending stiffness	EI = $4.26*10^7$ kNm^2
Span length	L = 30 m
Analyzed velocities of the moving force groups	Case a): v_1 = 50 m/s = 180 km/h
	Case a): v_2 = 100 m/s = 360 km/h
	Case b): v_1 = 100 m/s = 360 km/h
	Case b): v_2 = 120 m/s = 432 km/h
Critical damping ratio	ξ= 1.5 % = 0.015
Path length	L_p = 500 m

In this calculation we neglect the mass effect of the moving force, because based on Ref. [4] we have benchmark results to compare our FEM-Design results. In this example the structure is the same as in the previous example, only the moving load differs from it.

In this example we considered two different load groups as the moving loads. Fig. 16 top shows case a) and bottom shows case b). In both cases we modeled 12 motor trains group as the moving load. Case a) includes 12 individual concentrated loads (12 x 544 kN) to consider the weights of the train cars. In case b) we considered that every train cars have 4 axes, thus the load group contains 12 x 4 x 136 kN. The resultants of the load groups are the same.

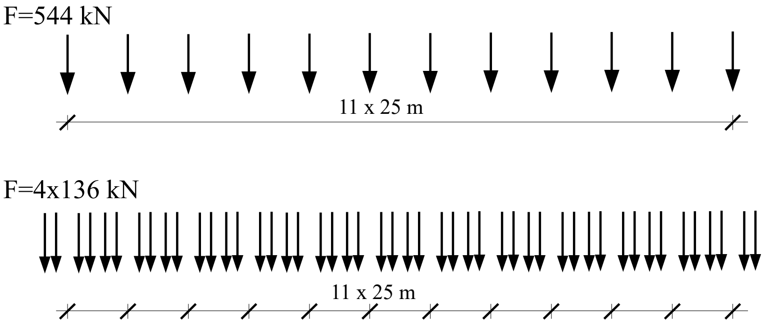

Figure 16 – The considered load groups to the dynamic analysis

Thus in these examples case a) and case b) load groups will be move on the path.

In this example we considered damping based on the equivalent Kelvin-Voigt damping.

Considering the given critical damping ratio and the first eigenfrequency of the given structure (based on the previous example) the Rayleigh damping parameters are as follows:

$$\alpha=0; \quad \beta=\frac{\xi}{\pi f_1}=\frac{0.015}{\pi 3.997}=0.001195$$

With the given input parameters Fig. 17 shows the dynamic response of the mid-span in function of the load position based on Ref. [4] in case a) load groups with v_1 = 50 m/s = 180 km/h and v_2 = 100 m/s = 360 km/h.

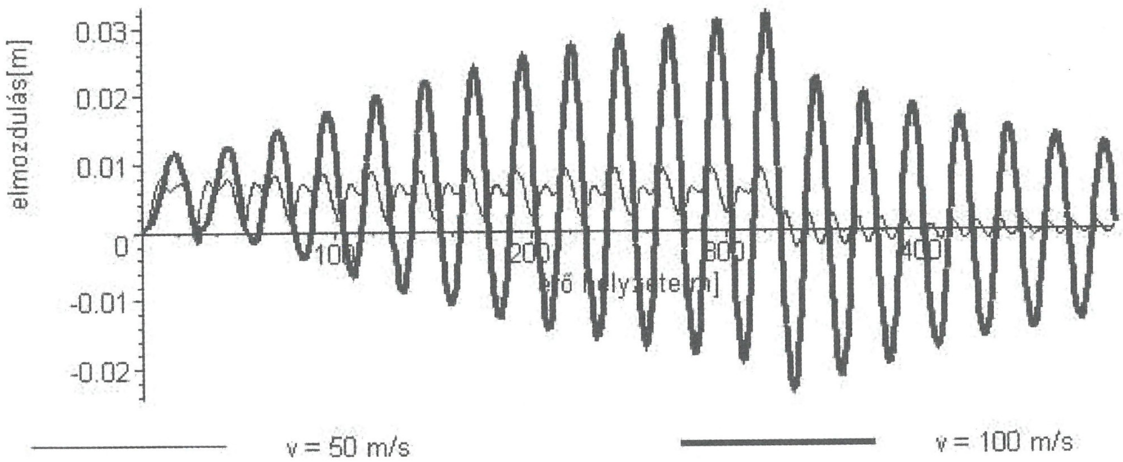

Figure 17 – The dynamic deflection of the mid-span based on Ref. [4] *in case a) with the two velocities*

Fig. 18-19 shows the dynamic response of mid-span in function of the load positions based on the FEM-Design moving load dynamic calculation in case a) load group.

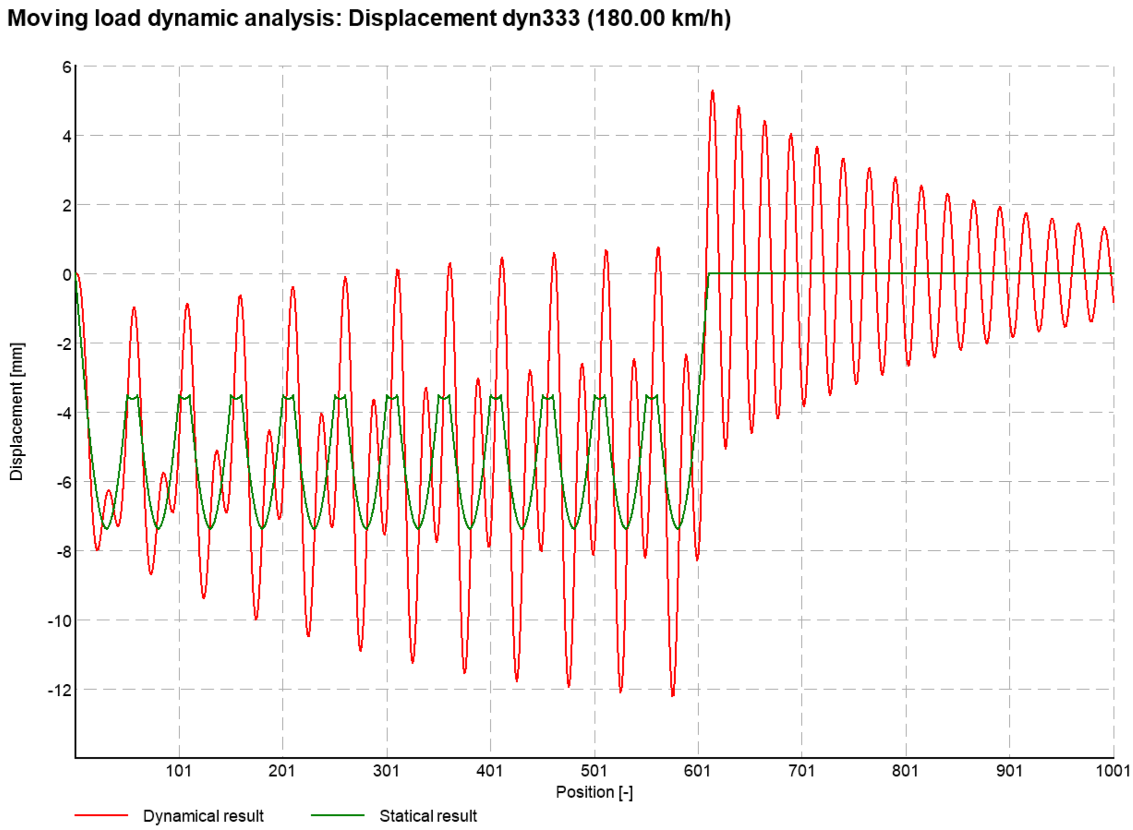

Figure 18 – The dynamic and static displacements of the mid-span in function of force positions with v_1 velocity in FEM-Design in case a)

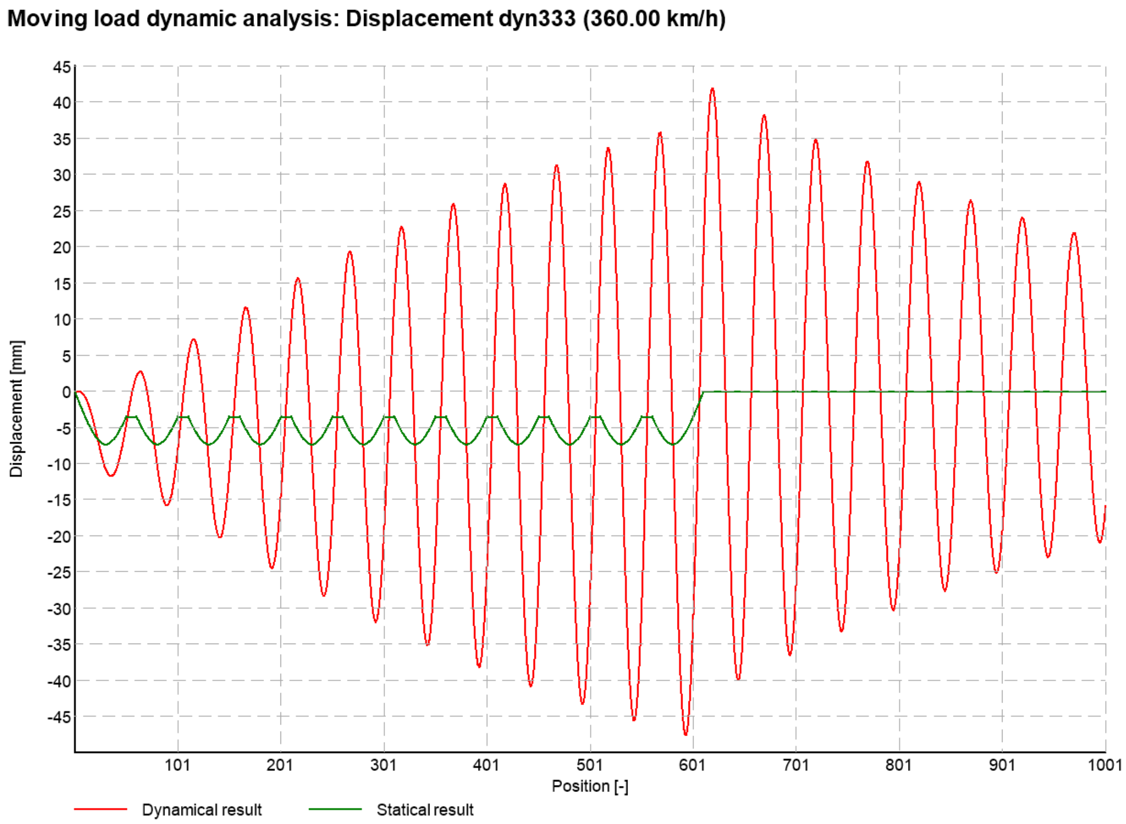

Figure 19 – The dynamic and static displacements of the mid-span in function of force positions with v_2 velocity in FEM-Design in case a)

We can say that the results are very close to each other.

Fig. 20 shows the dynamic response of the mid-span in function of the load position based on Ref. [4] in case b) load groups with $v_1 = 100$ m/s = 360 km/h and $v_2 = 120$ m/s = 432 km/h.

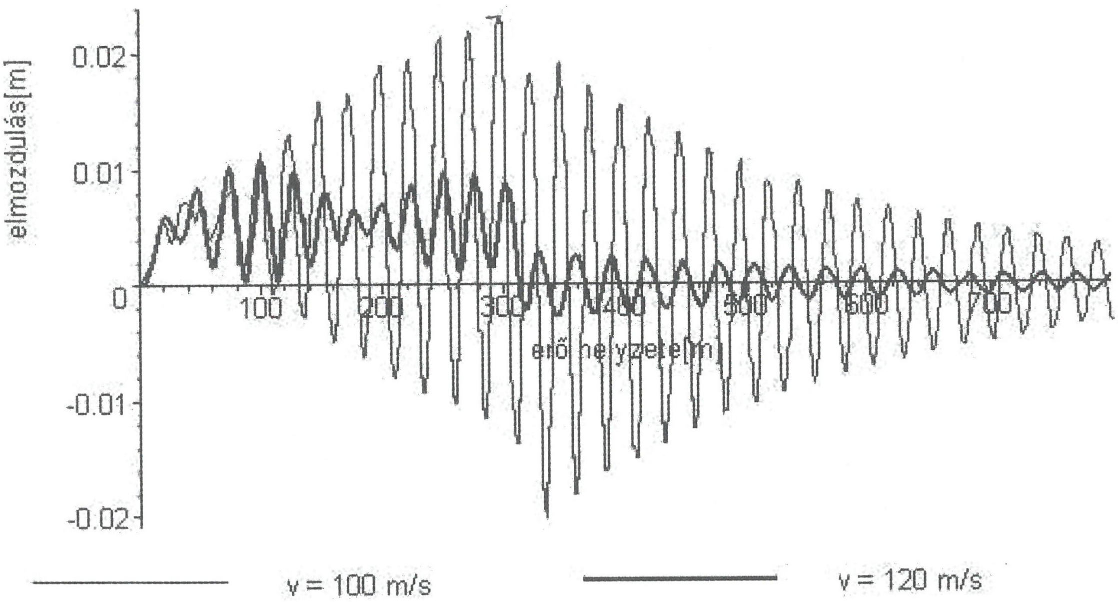

Figure 20 – The dynamic deflection of the mid-span based on Ref. [4] *in case b) with the two velocities*

Fig. 21-22 shows the dynamic response of mid-span in function of the load positions based on the FEM-Design moving load dynamic calculation in case b) load groups.

We can say that the results are very close to each other.

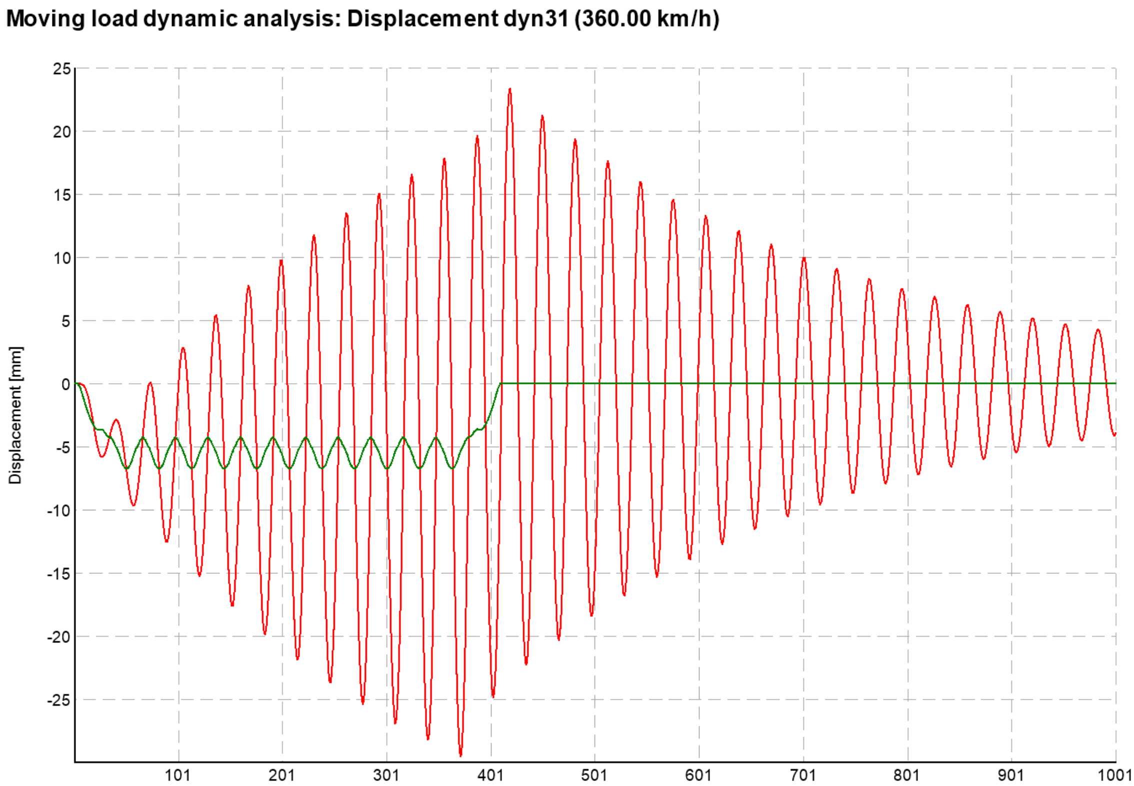

Figure 21 – The dynamic and static displacements of the mid-span in function of force positions with v_1 velocity in FEM-Design in case b)

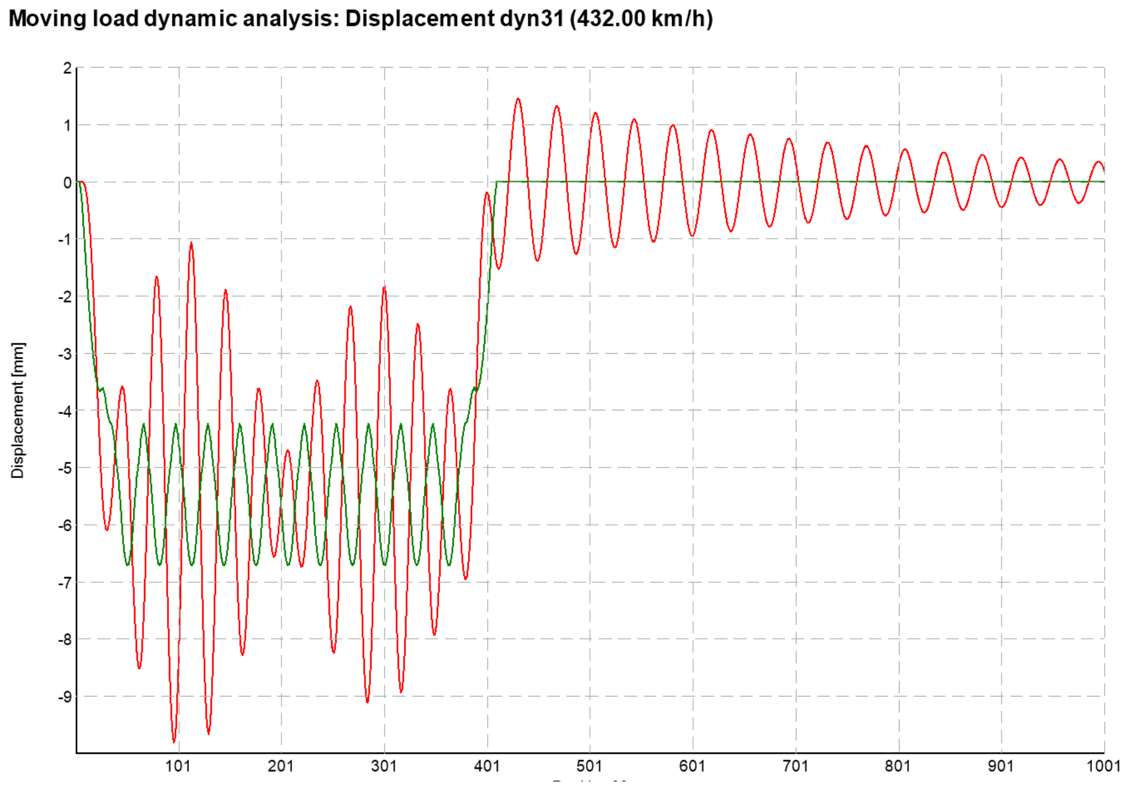

Figure 22 – The dynamic and static displacements of the mid-span in function of force positions with v_2 velocity in FEM-Design in case b)

Download link to the example file:
Case a):
http://download.strusoft.com/FEM-Design/inst190x/models/8.7 Dynamic response of a moving force group on a simply supported beam case a.str

Case b):
http://download.strusoft.com/FEM-Design/inst190x/models/8.7 Dynamic response of a moving force group on a simply supported beam case b.str

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[1] Chopra A.K., Dynamics of Structures, Prentice Hall, Fourth edition, 2011.

[2] Bathe K.J., Finite element procedures, Second Edition, Prentice Hall, 2016.

[3] Chatterjee A., Vaidya T.S., Dynamic Analysis of Beam under the Moving Mass for Damage Assessment, International Journal of Engineering Research & Technology, Vol. 4, 788-796., 2015.

[4] Györgyi J., Structural Dynamics (in Hungarian), Műegyetemi kiadó, 2006.

Notes