FEM-Design

Footfall Analysis

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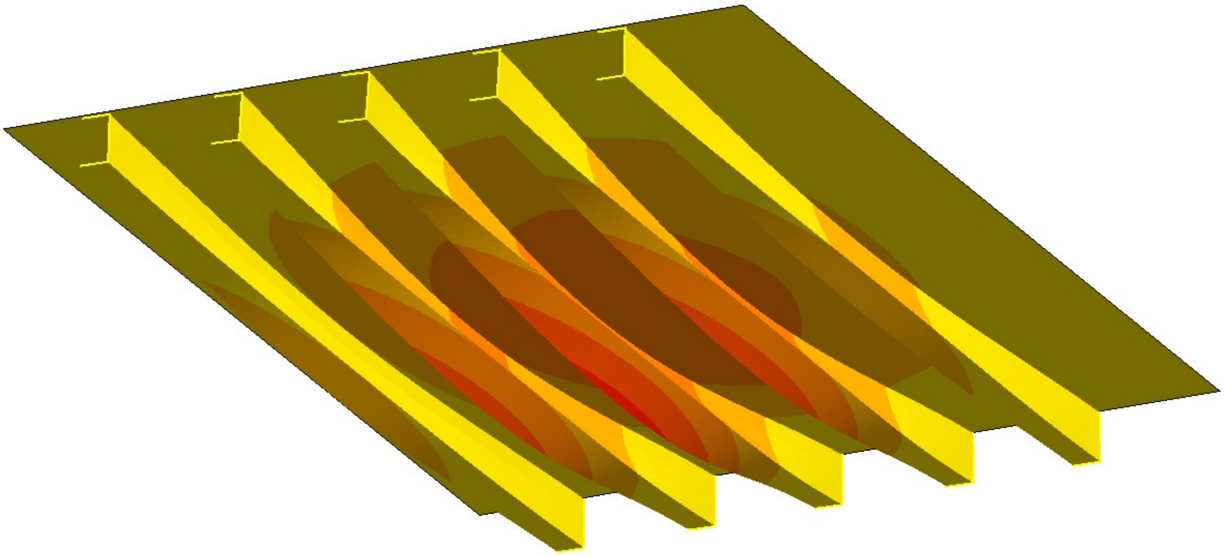

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Footfall Analysis

Trademarks

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List of symbols

Scalars

$a_{w,RMS,e}$	vertical weighted RMS acceleration at point e from excitation at point e
$a_{w,RMS,e,r}$	vertical weighted RMS acceleration at point r from excitation at point e
f_p	excitation frequency
f_n	eigenfrequency of the n-th eigenshape
h	the number of the excitation harmonic under consideration
n	the number of the eigenvector under consideration
n_e	effective number of people
t	time
v	velocity of walking
$D_{n,h}$	dynamic magnification factor for accelerations by the n-th mode shape for the the h-th harmonic
D_n	square-root sum of squares of the dynamic magnification factor for displacements by the n-th mode shape
F_{In}	impulsive force for the n-th mode shape
F_h	the excitation force amplitude for the h-th Fourier harmonic
L_p	length of the walking path
M_n	modal mass of the n-th mode shape (in FEM-Design the eigenvectors are normalized to the mass matrix, therefore the modal mass is always 1 tonne)
$N_{footstep}$	is the number of footsteps.
Q	the static force exerted by an average person
R	response factor at a point
W	weighting factor
α_h	h-th Fourier coefficient
δ	logarithmic decrement
$\mu_{e,n}$	vertical translational value of the n-th mode shape vector at the excitation point
$\mu_{r,n}$	vertical translational value of the n-th mode shape vector at the response point
ζ	critical damping ratio
ρ	resonance build-up factor

Vectors

$\boldsymbol{a}_{RMS}$	RMS acceleration vector
$\boldsymbol{\mu}_n$	the eigenvector of the n-th mode shape
$\boldsymbol{f}_h$	the excitation force amplitude vector for the h-th Fourier harmonic
$\boldsymbol{q}$	the load vector for the rhythmic crowd load from the excitation load case

Abbreviations

RMS	root-mean-square

1 Footfall analysis

1.1 Theoretical background

By a footfall induced vibration of a structure (e.g. floors) the dynamic response can be split into two parts. These two parts are the transient and the steady-state vibrations (see Ref. [1][2][4]). If the structure is relatively stiff then the transient response is more significant than the steady-state, but if the structure is less stiff then the steady-state response is remarkable and the transient part is negligible.

If the transient response is dominant then the applied excitation force will behave a series of impulses instead of a continuous function. The transient vibration can be understood as a series of damped free vibrational response of the system therefore the response mainly depend on the properties of the structure (e.g. eigenfrequency, mass and damping) and not from the frequency of the excitation force. Fig. 1 shows a typical time-acceleration diagram of a transient vibration.

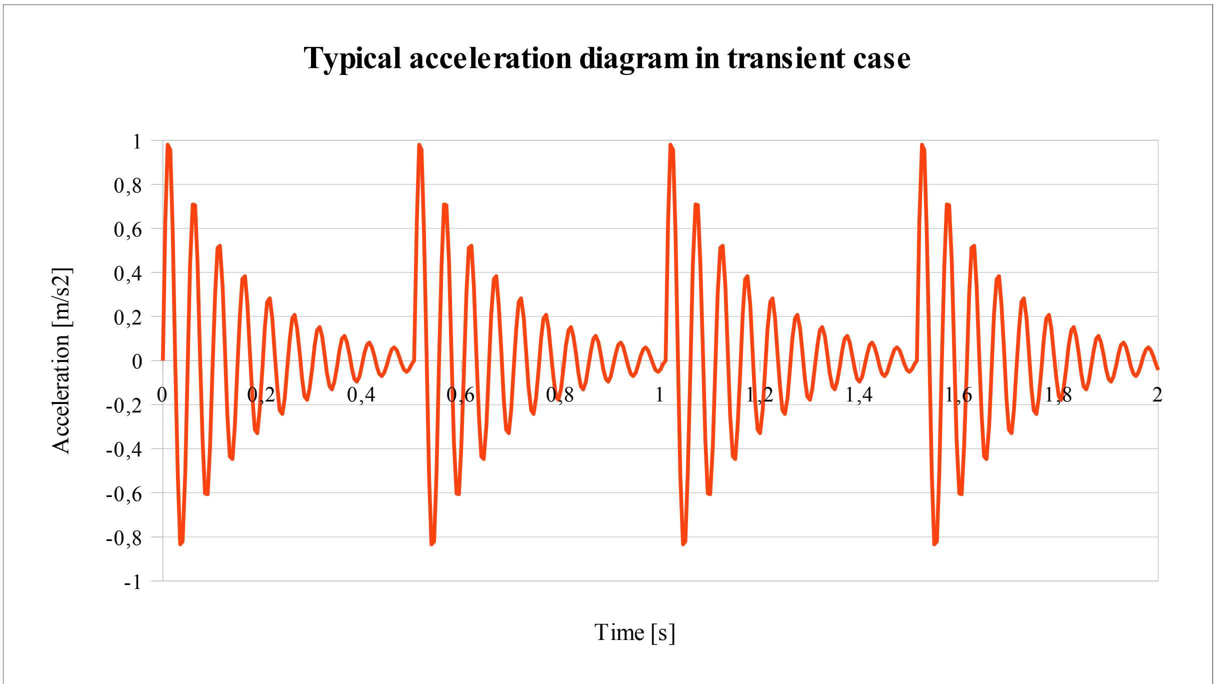


Figure 1 – Typical time-acceleration diagram by a transient dominant vibration

The steady-state response is when the wavefront has settled down. In this case resonance can appear if one of the eigenfrequencies of the structure is equal to the excitation frequency or its harmonics (integer multiple of the excitation frequency due to the Fourier series as harmonic excitation force). In this case the transient solution is negligible beside the steady-state solution and the amplitude of the acceleration becomes constant after a while (see Fig. 2) thus time is need for the evolution of the steady-state amplitude. Therefore if a walking path is sufficiently short a steady-state resonance condition may not be reached.

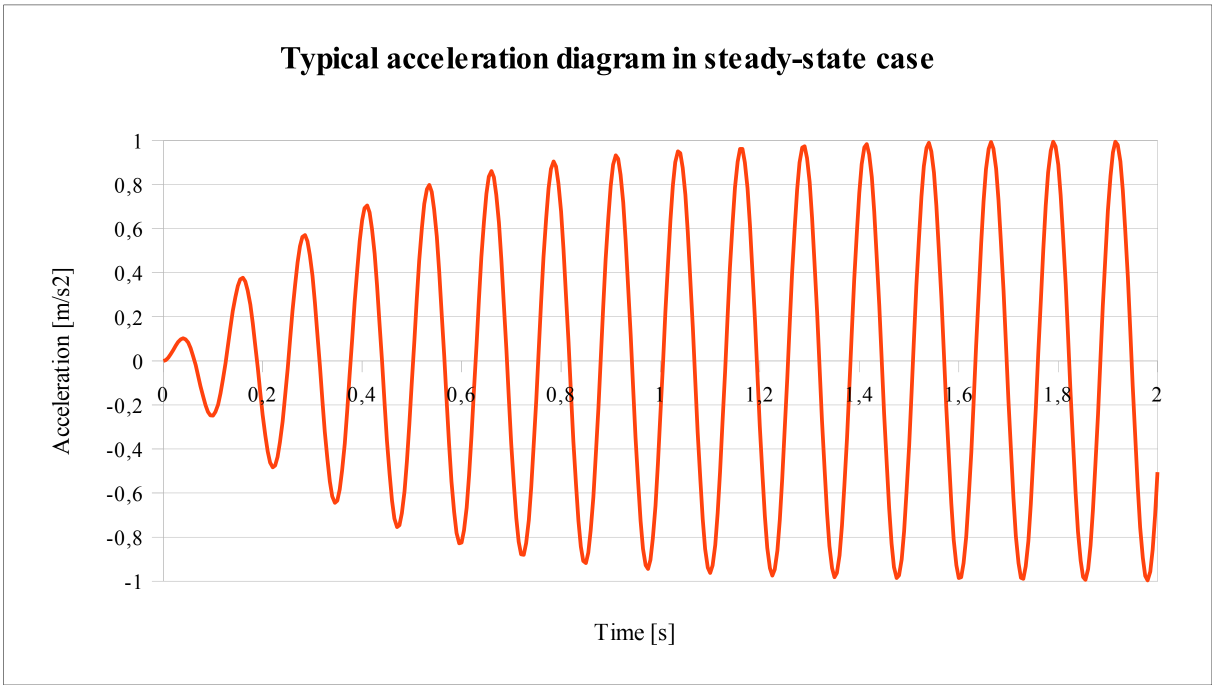


Figure 2 – Typical time-acceleration diagram by a steady-state dominant vibration

In FEM-Design the main purpose is to calculate the vibration behaviour of the floors (mainly accelerations, response factors and dynamic amplification factors). There are different calculation methods regarding the type of excitation forces and standardizations.

There are several basic assumptions in FEM-Design by the footfall analysis. According to these assumptions the users should consider and adjust some parameters before the calculations to get appropriate results:

- By the necessary eigenfrequency/vibration shape calculation only the vertical masses will be considered to avoid the not relevant shapes to the task. The excitation force is vertical and the response is dominantly also vertical.
- The excitation frequency is in a range (thanks to the phenomenon, there are a minimum frequency and a maximum frequency value of the considered walking frequency as excitation force). Thus the calculations should be done with several excitation frequencies in the mentioned interval and get the most unfavourable results. Frequency steps give the number of the considered excitation frequencies in the given interval.
- Calculations will be performed with the eigenfrequencies which are under the cut-off eigenfrequency. To avoid the numerical contradictions the last of the considered eigenfrequencies will always be greater than the cut-off eigenfrequency.
- During the footfall analysis calculation the considered damping is constant and it is given with a ratio between the damping and the critical damping in [%]. This is the critical damping ratio (ζ). In some references (e.g. Ref. [3]) the logarithmic decrement (δ) is given as damping parameter. The mathematical connection between the logarithmic decrement and critical damping ratio: $\delta = 2\,\pi\,\zeta$.

There are several ways to present the acceleration of a system. The most obvious is the largest acceleration. However, this gives no indication as to the amount of time the system is subjected to this level of acceleration. Instead of the largest acceleration the root-mean-square (RMS) acceleration is widely used. The RMS acceleration is calculated as follows:

$$a_{RMS}=\sqrt{\frac{1}{T}\int_0^T a(t)^2\,dt} \quad , \qquad \text{(Eq. 1)}$$

where T is the period under consideration, $a(t)$ is the acceleration function and t is time.

The response factor of a floor (according to Ref. [2]) is the ratio between the calculated weighted RMS acceleration from either the steady-state or transient methods and the 'base value' given in BS 6472. The vibration response is considered to be satisfactory for continuous vibrations when the calculated response does not exceed a limiting value appropriate to the environment (which is expressed in BS 6472 and ISO 10137 as a multiplying factor. For vertical vibrations the response factor R is given by:

$$R=\frac{a_{RMS}}{0.005} \qquad \text{(Eq. 2)}$$

The response factor is a unitless number and it is calculated in the same way by the different excitation methods.

In FEM-Design there are three different excitation methods:

- Self excitation
- Full excitation
- Rhythmic crowd load

You can find the details about these methods below.

1.2 Different excitation methods

1.2.1 Self excitation

In this case the excitation force is one concentrated vertical force. Self excitation analyses the response in the same node to which the excitation force is applied. It means that the excitation is in the same node where we calculate the response (*e*=*r*). For example thus it means that the concentrated excitation force is applied at node number 1 and the response is calculated here also. The response in node number 2 comes from the exicitation force which was applied on node number 2 and so on (see Fig. 3). This method is usually proper for walking activities and response analysis according to Ref. [2].

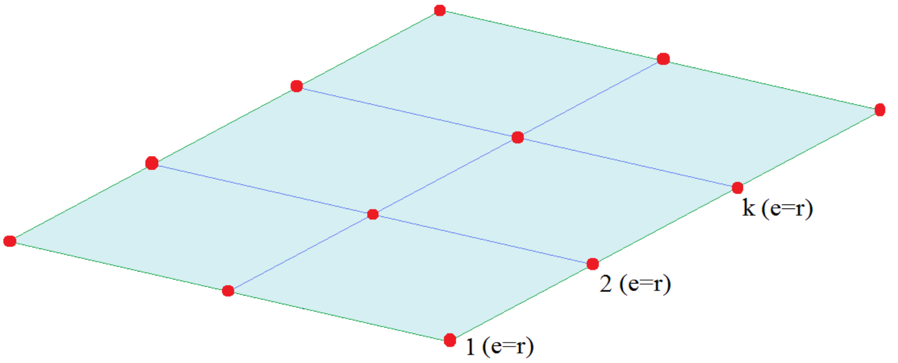


Figure 3 – The self excitation method: the response point equal to the excitation point

Here the user should select region(s) where the excitations and responses will be calculated/evaluated. The results will be available on the different selected region(s) individually.

The user should adjust the number of footsteps, the mass of the walker (it is usually around 76 kg), the frequency weighting curve (see Subchapter 1.3) and the Fourier coefficients (see Subchapter 1.4).

1.2.1.1 Steady-state accelerations

According to the informations in the introduction one of the accelerations which should be calculated is the steady-state RMS acceleration. The steady-state weighted vertical RMS acceleration in a node where the excitation force is considered comes from the following equation:

$$a_{w,e,RMS[steady\,state]} = \rho \frac{1}{\sqrt{2}} \sqrt{\sum_{h=1}^{H} \left(\sum_{n=1}^{N} \left(\mu_{e,n}^2 \frac{F_h}{M_n} D_{n,h} W_h \right) \right)^2} \quad , \qquad \text{(Eq. 3)}$$

where:

- H is the total number of the considered harmonics.
- N is the total number of the considered eigenfrequencies.
- $\mu_{e,n}$ is the vertical translational value of the n^{th} mode shape vector at excitation point.
- $F_h = \alpha_h Q$ is the amplitude of the excitation force for the h^{th} harmonic.
- α_h is the Fourier coefficient for the h^{th} harmonic, see Subchapter 1.4 about this.
- Q is the static force exerted by an average person (normally taken as 0.746 kN).
- M_n is the modal mass of modeshape n and it is equal to 1[t] if the mode shape vector is normalized to the mass matrix. In FEM-Design the mode shapes are always normalized to the mass matrix.
- $D_{n,h} = \dfrac{h^2\left(\dfrac{f_p}{f_n}\right)^2}{\sqrt{\left(1-h^2\left(\dfrac{f_p}{f_n}\right)^2\right)^2 + 4\zeta^2 h^2\left(\dfrac{f_p}{f_n}\right)^2}}$ is the magnification factor for the accelerations.
- h is the number of the h^{th} harmonic under consideration.
- f_p is the excitation frequency.
- f_n is the eigenfrequency of the n^{th} mode shape.
- ζ is the critical damping ratio.
- W_h is the weighting factor calculated with the frequency of the harmonic under consideration hf_p, see Subchapter 1.3 also.
- $\rho = 1 - e^{\left(\frac{-2\pi\zeta L_p f_p}{v}\right)}$ is the resonance build-up factor.

  If the walking path is sufficiently short a steady-state condition may not be reached thus this reduction factor will be used. If the damping is exactly set to 0 then ρ = 1.0 and the program neglects this effect.
- $v = 1.67 f_p^2 - 4.83 f_p + 4.50$ is the velocity of walking.

  If the adjusted f_p value is less than 1.7 Hz then this velocity is calculated with f_p = 1.7 Hz. And if the adjusted f_p value is greater than 2.4 Hz then this velocity is calculated with f_p = 2.4 Hz.
- $L_p = 0.75\,\text{m} \cdot N_{footsteps}$ is the length of the walking path.
- $N_{footsteps}$ is the number of footsteps.

1.2.1.2 Transient accelerations

The weighted vertical acceleration function of the transient part of the vibration in a node where the excitation force is considered:

$$a_{w,e[transient]}(t)=\sum_{n=1}^{N} 2\pi f_n \sqrt{1-\zeta^2}\,\mu_{e,n}^2 \frac{F_{In}}{M_n} \sin\left(2\pi f_n \sqrt{1-\zeta^2}\,t\right) e^{-\zeta 2\pi f_n t} W_n \quad , \qquad \text{(Eq. 4)}$$

where, in addition to those described above:

- $F_{In}=60\dfrac{f_p^{1.43}}{f_n^{1.3}}\dfrac{Q}{700}$ is the excitation impulse force of the n^{th} mode shape.
- W_n is the weighting factor calculated with the eigenfrequency of n^{th} mode shape. See Subchapter 1.3 also.

Based on the transient acceleration function the RMS acceleration will be calculated with the following formula:

$$a_{w,RMS,e[transient]}=\sqrt{f_p \int_0^{1/f_p} a_{w,e[transient]}(t)^2\,dt} \qquad \text{(Eq. 5)}$$

1.2.1.3 Final acceleration of one node

The overall vertical RMS acceleration result of one node will be the greater value from the steady-state and transient response:

$$a_{w,RMS,e}=\max\left[a_{w,RMS,e[transient]};a_{w,RMS,e[steady\ state]}\right] \qquad \text{(Eq. 6)}$$

The response factor value of one node is calculated based on Eq. 2 with this acceleration value.

1.2.2 Full excitation

In this case the excitation force is one concentrated vertical force. Full excitation analyses the response in any node, to effect of the force applied to another node. In this case the excitation force is applied independently in the selected node or nodes and there is no interaction between them. The user can adjust one or more excitation points and FEM-Design calculates the responses in all nodes individually ($e \neq r$). For example thus it means that if the excitation force is applied at node number 2 then the response will be calculated in all nodes based on this excitation force at node number 2 (see Fig. 4). This method is usually proper for walking activities and response analysis according to Reference [2].

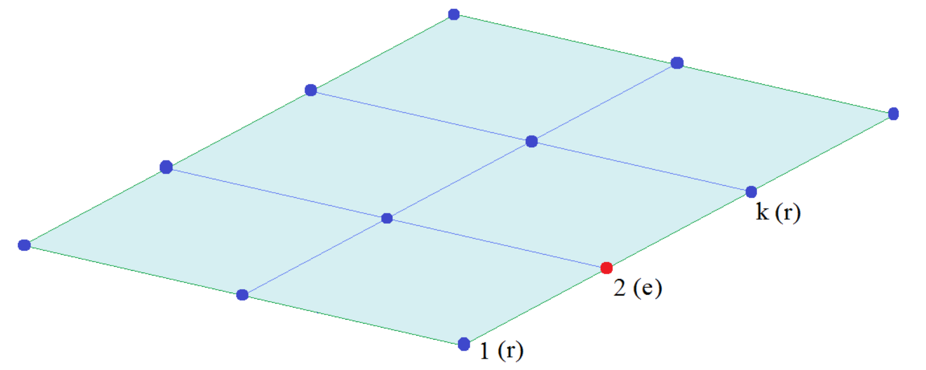


Figure 4 – The full excitation method: the excitation point is at node number 2 and the response is calculated in all nodes based on this excitation force position

Here the user should select excitation point(s) where the excitation force(s) will be applied individually. The response will be calculated in all nodes of the structure. The results will be available for the different excitation point(s) individually.

The user should adjust the number of footsteps, the mass of the walker (it is usually around 76 kg) and the frequency weighting curve (see Subchapter 1.3) and the Fourier coefficients (see Subchapter 1.4).

1.2.2.1 Steady-state accelerations

The steady-state weighted vertical RMS acceleration in a node where the response is calculated comes from the following equation:

$$a_{w,RMS,e,r[steady\ state]} = \rho \frac{1}{\sqrt{2}} \sqrt{\sum_{h=1}^{H} \left(\sum_{n=1}^{N} \left(\mu_{e,n} \mu_{r,n} \frac{F_h}{M_n} D_{n,h} W_h \right) \right)^2} \quad , \qquad \text{(Eq. 7)}$$

where, in addition to those described above:

– $\mu_{r,n}$ is the vertical translational value of the n^{th} mode shape vector at the point where the response is to be calculated.

1.2.2.2 Transient accelerations

The weighted vertical acceleration function of the transient part of the vibration in a node where the response is to be calculated comes from the following equation:

$$a_{w,e,r[transient]}(t)=\sum_{n=1}^{N} 2\pi f_n\sqrt{1-\xi^2}\,\mu_{e,n}\mu_{r,n}\frac{F_{In}}{M_n}\sin\left(2\pi f_n\sqrt{1-\xi^2}\,t\right)e^{-\xi 2\pi f_n t}W_n \quad , \qquad \text{(Eq. 8)}$$

Based on the transient acceleration function the RMS acceleration will be calculated with the following formula:

$$a_{w,RMS,e,r[transient]}=\sqrt{f_p\int_0^{1/f_p} a_{w,e,r[transient]}(t)^2\,dt} \qquad \text{(Eq. 9)}$$

1.2.2.3 Final acceleration of one response node

The overall vertical RMS acceleration result of one response node will be the greater value from the steady-state and transient response:

$$a_{w,RMS,e,r}=\max\left[a_{w,RMS,e,r[transient]};a_{w,RMS,e,r[steady\,state]}\right] \qquad \text{(Eq. 10)}$$

The response factor value of one reponse node is calculated based on Eq. 2 with this acceleration value.

1.2.3 Rhythmic crowd load

In this case the excitation force can be distributed surface vertical force (or in some situation concentrated or line distributed force). This type of excitation is the so-called synchronised crowd activities. This type of excitation force is usually proper for small groups induced vibrations (e.g. dance and aerobic areas) according to Ref. [2][3]. The user should make load case(s) (typically with vertical distributed surface load) which will represent the static load(s) of the crowd group(s). During the footfall analysis only the vertical components of the force-a-like values in the selected load case(s) will be considered. The results will be available for the different load case(s) individually and will show the accelerations in all nodes of the structure as response nodes. For example see Fig. 5 where the load vector of the distributed surface load will represent the static load of the crowd group and the program calculates the response in all nodes based on the consideration of the dynamic effect of this crowd load.

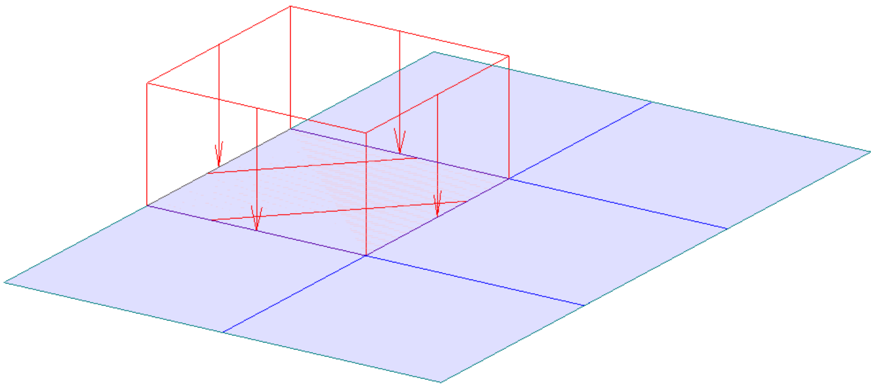


Figure 5 – The rhythmic crowd load method: the excitation force is the distributed surface load and the response is calculated in all nodes

Here the user should select load case(s) which will include the static load of the crowd group. The user should adjust the Fourier coefficients also (see Subchapter 1.4).

In this excitation method only the steady-state part of the solution will be evaluated with the following general vector equation which will represent the RMS acceleration response vector of the structure. In Eq. 11 the squaring by the summation and the square root should be interpreted individually by the elements of the vector):

$$\boldsymbol{a}_{RMS}=\frac{1}{\sqrt{2}}\sqrt{\sum_{h=1}^{H}\left(\sum_{n=1}^{N}\left(\boldsymbol{\mu}_n\cdot\boldsymbol{\mu}_n^{\mathrm{T}}\cdot\boldsymbol{f}_h\cdot D_{n,h}\right)\right)^2}\quad, \qquad \text{(Eq. 11)}$$

where, in addition to those described above:

- $\boldsymbol{\mu}_n$ is the eigenvector of the n^{th} mode shape (normalized to the mass matrix).
- $\boldsymbol{f}_h=\alpha_h\boldsymbol{q}$ is the amplitude of the excitation force vector for the h^{th} harmonic.

– $\boldsymbol{q}$ is the load vector based on the selected load case as excitation load.

The response factor values are calculated based on Eq. 2 with this RMS acceleration vector.

Another useful result by this excitation method is the dynamic magnification factor for displacements which will be calculated by the n^{th} mode shape as follows:

$$D_n = \sqrt{\sum_{h=1}^{H} \left(\frac{\alpha_h}{\sqrt{\left(1 - h^2 \left(\frac{f_p}{f_n}\right)^2\right)^2 + 4\xi^2 h^2 \left(\frac{f_p}{f_n}\right)^2}} \right)^2} \qquad \text{(Eq. 12)}$$

By the results the user can see the maximum values of these magnification factors regarding the most unfavourable excitation frequencies.

With this value the users can give the equivalent static load. The static deflections under this equivalent static load will be equal to the maximum displacements of the dynamic behaviour of the structure under the rhythmic crowd load.

$$\boldsymbol{q}_{equivalent} = (1 + D_n)\boldsymbol{q} \qquad \text{(Eq. 13)}$$

1.3 Weighting factors

In FEM-Design there are two different options to set the required weighting factors for the self and full excitation method.

It is for the consideration of the different response of the human body in different room types and situations, see Ref. [1].

Z-axis (vertical) vibration W_g weighting curve (BS 6841):

$W = 0.5\sqrt{f}$	for $1\,\text{Hz} < f < 4\,\text{Hz}$
$W = 1.0$	for $4\,\text{Hz} \leq f \leq 8\,\text{Hz}$
$W = 8.0/f$	for $f > 8\,\text{Hz}$

Z-axis (vertical) vibration W_b weighting curve (BS 6841):

$W = 0.4$	for $1\,\text{Hz} < f < 2\,\text{Hz}$
$W = f/5.0$	for $2\,\text{Hz} \leq f < 5\,\text{Hz}$
$W = 1.0$	for $5\,\text{Hz} \leq f \leq 16\,\text{Hz}$
$W = 16.0/f$	for $f > 16\,\text{Hz}$

1.4 The types of the adjustable Fourier coefficients

In FEM-Design there are five different options to set the required Fourier coefficients for the steady-state calculations. The α_h Fourier coefficients are as follows:

1.4.1 User defined coefficients

The user can apply maximum six arbitrary Fourier harmonic coefficients by hand.

1.4.2 SCI P354 Table 3.1 based on Reference [2]

Harmonic h	Excitation frequency range hf_p (Hz)	Design value of coefficient α_h
1	1.8 to 2.2	$0.436(hf_p - 0.95)$
2	3.6 to 4.4	$0.006(hf_p + 12.3)$
3	5.4 to 6.6	$0.007(hf_p + 5.2)$
4	7.2 to 8.8	$0.007(hf_p + 2.0)$

1.4.3 SCI P354 Equation 20 based on Reference [2]

Harmonic h	Design value of coefficient α_h
1	$1.61p^{-0.082}$
2	$0.94p^{-0.24}$
3	$0.44p^{-0.31}$
Where p is the number of participants in the rhythmic activity ($2 \leq p \leq 64$).	

1.4.4 Concrete Center Table 4.3 based on Reference [4]

Harmonic h	Excitation frequency range hf_p (Hz)	Design value of coefficient α_h
1	1.0 to 2.8	$\min(0.41(hf_p - 0.95)\,;\,0.56)$
2	2.0 to 5.6	$0.069 + 0.0056hf_p$
3	3.0 to 8.4	$0.033 + 0.0064hf_p$
4	4.0 to 11.2	$0.013 + 0.0065hf_p$

1.4.5 Danish Annex C based on Reference [3]

The coefficients contain the size reduction factor!

Possible to move about freely	
Harmonic ***h***	**Design value of coefficient** α_h
1	1.6
2	$1.0\sqrt{0.3+(1-0.3)\frac{1}{n_e}}$
3	$0.2\sqrt{0.03+(1-0.03)\frac{1}{n_e}}$
Where n_e is the effective number of people.	

Reduced possibility to move about	
Harmonic ***h***	**Design value of coefficient** α_h
1	0.40
2	$0.25\sqrt{0.1+(1-0.1)\frac{1}{n_e}}$
3	$0.05\sqrt{0.01+(1-0.01)\frac{1}{n_e}}$
Where n_e is the effective number of people.	

Walking	
Harmonic ***h***	**Design value of coefficient** α_h
1	$0.40\sqrt{\frac{1}{n_e}}$
2	$0.10\sqrt{\frac{1}{n_e}}$
3	$0.06\sqrt{\frac{1}{n_e}}$
Where n_e is the effective number of people.	

2 Verification examples

2.1 Footfall analysis of a concrete footbridge

Example taken from Ref. [4]. Let's take the following footbridge statical system from Fig. 6.

Inputs for the self excitation footfall analysis:

Dynamic elastic modulus of concrete	E = 38 GPa
The distributed load (load-mass conversion)	p = 18.13 kN/m
Number of considered mode shapes	N = 3
Inertia of the cross-section	$I = 0.056\ m^4$
Area of the section	$A = 0.77\ m^2$
Number of footsteps (conservative estimation)	$N_{footstep} = 100$ pcs
Mass of the walker	m = 71.36 kg
Frequency weighting curve	W_g
The excitation frequency interval	$f_{p,min}$= 1 Hz, $f_{p,max}$= 2.8 Hz
Frequency steps	steps = 100 pcs
The cut-off eigenfrequency	f_{cut} = 15 Hz
Damping	ζ= 1.5 %
Fourier coefficients (see Subchapter 1.4)	The Concrete Centre Table 4.3

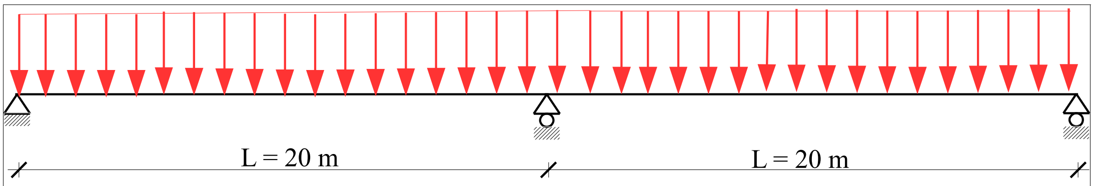


Figure 6 – The concrete footbridge with the considered load-mass conversion

The model is divided into 16 finite bar elements. The given distributed load is converted to mass with 1.0 factor (1848 kg/m) for the eigenfrequency calculation. The statical system is a beam with the given stiffness parameters and with 3 supports (see Fig. 6). All of the necessary parameters for the footfall analysis is given in the inputs. In FEM-Design the used excitation method was the self excitation method. For the self excitation method the adjusted region contained the full beam structure.

The first three mode shapes are visible in Fig. 7 based on the FEM-Design calculation. Table 1 contains the theoretical solutions about the eigenfrequencies of the first three modes according to Ref. [4] and FEM-Design results are also indicated. There are good agreements between the two results.

Mode	Theoretical (Hz)	FEM-Design (Hz)
1st	4.22	4.203
2nd	6.59	6.536
3rd	16.90	16.68

Table 1 – The first three eigenfrequencies

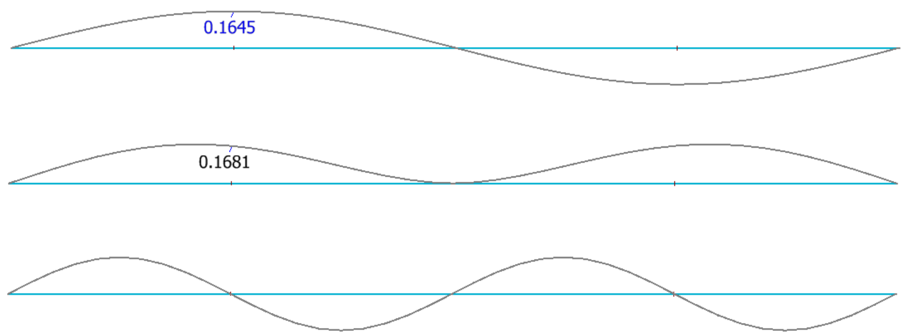


Figure 7 – The first three mode shapes [-]

This footbridge is relatively soft, therefore the steady-state acceleration will be greater than the transient. As a simple hand calculation check according to Eq. 3 the RMS acceleration for walking at 2.102 Hz is the following:

The amplitude of the excitation force by the second harmonic (see Subchapter 1.4):

$$F_2=\frac{71.36}{1000}\cdot 9.81\left(0.069+0.0056\cdot 2\cdot 2.102\right)=0.06478\,\mathrm{kN}$$

In this case the second harmonic of the excitation frequency causes resonance.

The dynamic magnification factor for the accelerations by the 1st mode shape and 2nd harmonic:

$$D_{1,2}=\frac{2^2\left(\frac{f_p}{f_1}\right)^2}{\sqrt{\left(1-2^2\left(\frac{f_p}{f_1}\right)^2\right)^2+4\zeta^2 2^2\left(\frac{f_p}{f_1}\right)^2}}=\frac{2^2\left(\frac{2.102}{4.203}\right)^2}{\sqrt{\left(1-2^2\left(\frac{2.102}{4.203}\right)^2\right)^2+4\cdot 0.015^2\cdot 2^2\left(\frac{2.102}{4.203}\right)^2}}=$$

$$=\frac{1}{\sqrt{0+4\cdot 0.015^2\cdot 1}}=\frac{1}{2\cdot 0.015}=33.33$$

Based on these values the RMS acceleration at mid-span (see Fig. 7 also):

$$a_{w,midspan,RMS[steady\ state]}=\frac{1}{\sqrt{2}}\mu_{midspan,1}^{2}\frac{F_{2}}{M_{1}}D_{1,2}W_{2}=\frac{1}{\sqrt{2}}0.1645^{2}\frac{0.06478}{1}33.33\cdot 1.0=0.04131\frac{\mathrm{m}}{\mathrm{s}^{2}}$$

In Ref. [4] the peak acceleration value is a_{peak} = 0.06 m/s^2, therefore the comparable RMS value is:

$$a_{RMS}=\frac{a_{peak}}{\sqrt{2}}=\frac{0.06}{\sqrt{2}}=0.04243\frac{\mathrm{m}}{\mathrm{s}^{2}}$$ and the response factor based on Ref. [4]: $\mathrm{R}=8.5$

Based on the FEM-Design calculation these two values are (see Fig. 8 as well):

$$a_{RMS,FEM}=0.0443\frac{\mathrm{m}}{\mathrm{s}^{2}}$$ and $\mathrm{R}=8.86$.

There are good agreements between the results. The difference comes from the fact that not only the first mode shape has effect on the accelerations however in Ref. [4] and the hand calculation here considered only the first mode.

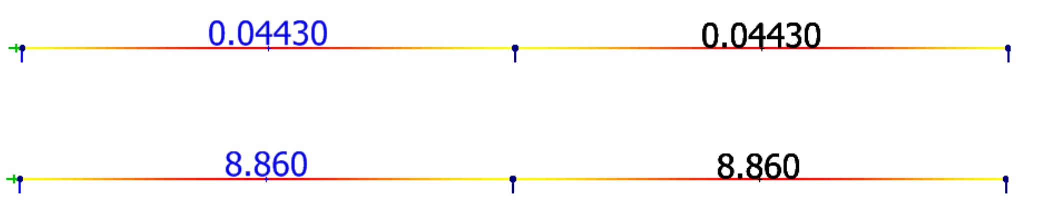


Figure 8 – The acceleration [m/s^2] *and the response factor* [-] *in FEM-Design*

Another interesting result could be the frequency curve. Fig. 9 shows the accelerations at the midspan point in function of the excitation frequencies. The red line is the steady-state response and the green one is the transient. Based on Fig. 9 we can say that in this example the transient response is really negligible compared to the steady-state response. The frequency curve clearly shows the resonance excitation frequencies where the peak RMS accelerations arise.

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/8.1 Footfall analysis of a concrete footbridge.str

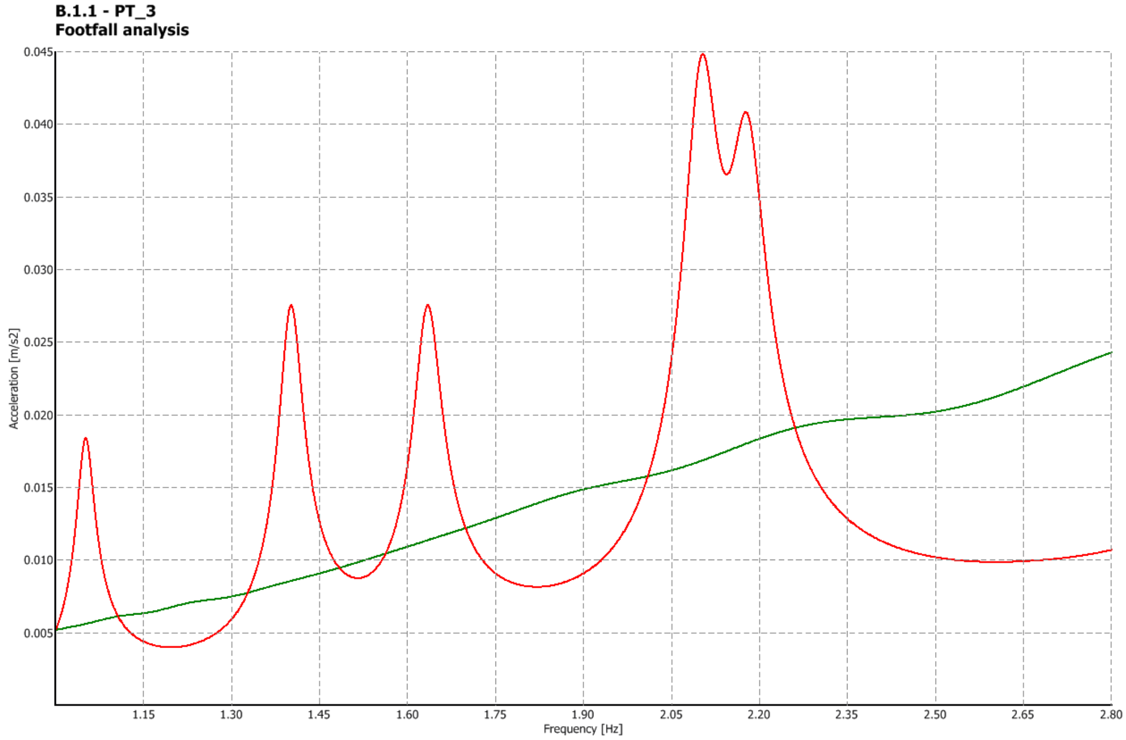


Figure 9 – Accelerations in function of excitation frequencies in FEM-Design
Red: steady-state response
Green: transient response

2.2 Footfall analysis of a composite floor

Example taken from Ref. [2]. Let's take a 130 mm deep normal weight concrete slab on top of 1.2 mm thick re-entrant deck. Slabs supported by 6.0 m span secondary beams at 2.48 m cross-centres which, in turn, are supported by 7.45 m span castellated primary beams in orthogonal direction, see Fig. 10. The input data and the geometry are available in Ref. [2].

Inputs for the self excitation footfall analysis:

Excitation region (see Fig. 10)	The whole floor slab
The distributed load (load-mass conversion)	$p = 4.48\ \mathrm{kN/m^2}$
Number of footsteps (conservative estimation)	$N_{footstep} = 100$ pcs
Mass of the walker	$m = 76$ kg
Frequency weighting curve	W_g
The excitation frequency interval	$f_{p,min} = 1.8$ Hz, $f_{p,max} = 2.2$ Hz
Frequency steps	steps = 100 pcs
The cut-off eigenfrequency	$f_{cut} = 15$ Hz
Damping	$\zeta = 4.68$ %
Fourier coefficients (see Subchapter 1.4)	SCI P354 Table 3.1

In Ref. [2] with the finite element calculation the first fundamental natural frequency was:

$f_1 = 10.80\,\mathrm{Hz}$

In Ref. [2] with the finite element calculation the response factor was:

$R = 3.18$

With the given parameters above and considering the geometry and the material properties based on Ref. [2] FEM-Design calculation gives the following results (see Fig. 10 also):

$f_{FEM} = 10.82\,\mathrm{Hz}$ and $R = 3.82$

We can say that there are good agreements between the results. However, it should be noted that in Ref. [2] the results of the calculation is given, but the details of the finite element model and calculation method is unclear, therefore there may be differences in the modeling methods. By this example it is very hard to say that the result in Ref. [2] is relevant because the hand calculation is quite different than the FEM calculation what was published in Ref. [2]. Based on our opinion the indicated FEM result in Ref. [2] belongs to the transient response as well as the result in FEM-Design.

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/8.2 Footfall analysis of a composite floor.str

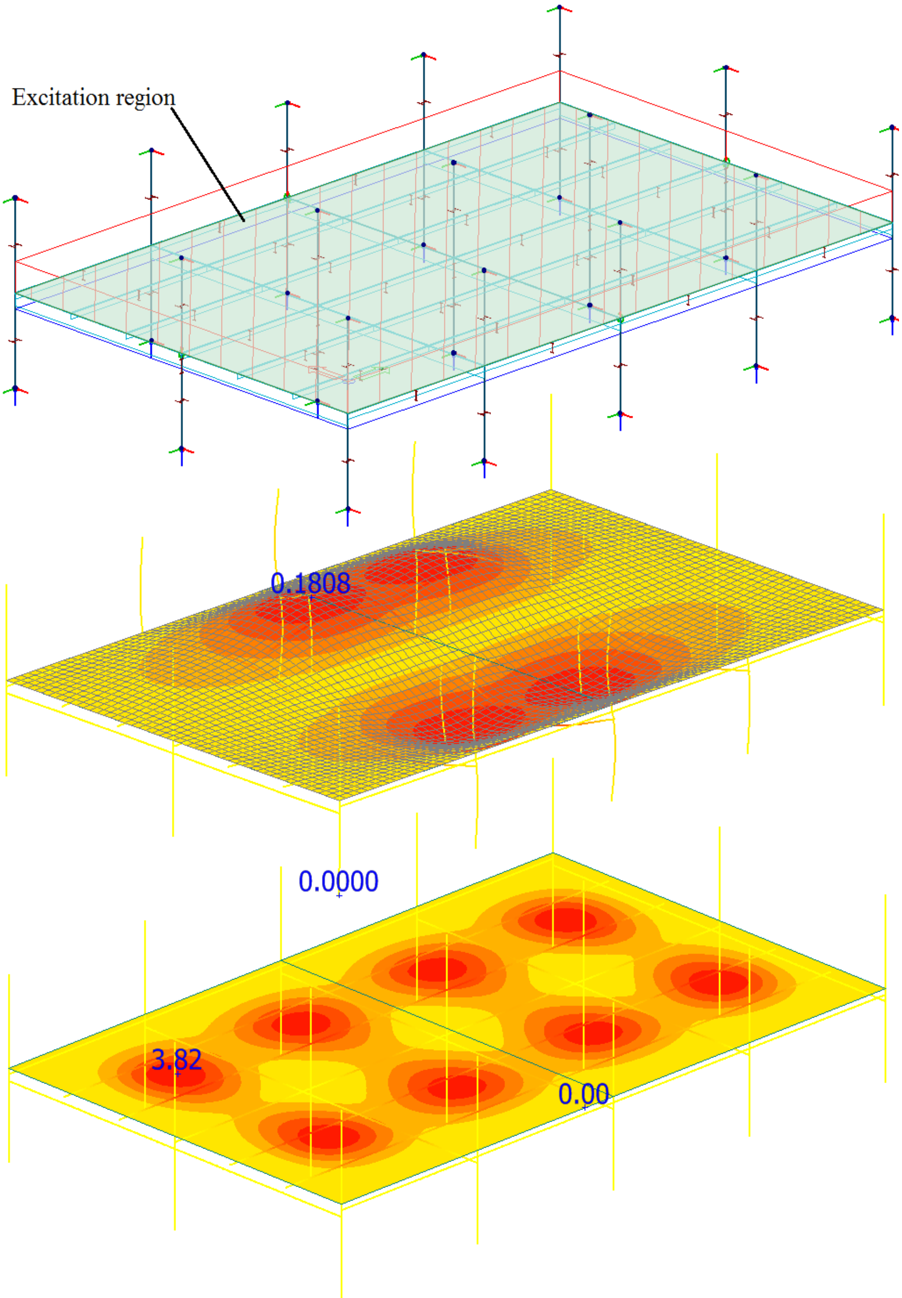


Figure 10 – The model, the first mode shape [-] *and the response factor* [-] *results in FEM-Design*

2.3 Footfall analysis of a lightweight floor

Example taken from Ref. [2]. Let's take a chipboard flooring on lightweight steel beams, see Fig. 11. The input data and the geometry are available in Ref. [2].

Inputs for the full excitation footfall analysis:

Excitation point (see Fig. 11)	In the middle of the floor
The distributed load (load-mass conversion)	$p = 0.69$ kN/m^2
Number of footsteps (conservative estimation)	$N_{footstep} = 100$ pcs
Mass of the walker	$m = 76$ kg
Frequency weighting curve	W_g
The excitation frequency interval	$f_{p,min} = 1.8$ Hz, $f_{p,max} = 2.2$ Hz
Frequency steps	steps = 100 pcs
The cut-off eigenfrequency	$f_{cut} = 15$ Hz
Damping	$\zeta = 5.0$ %
Fourier coefficients (see Subchapter 1.4)	SCI P354 Table 3.1

In Ref. [2] with the finite element calculation the first fundamental natural frequency was:

$$f_1 = 16.31\,\text{Hz}$$

In Ref. [2] with the finite element calculation the response factor was:

$$R = 53.9$$

In FEM-Design the average finite element size was 0.40 m. With the given parameters above and considering the geometry and the material properties based on Ref. [2] FEM-Design calculation gives the following results (see Fig. 11 also):

$f_{FEM} = 16.13\,\text{Hz}$ and $R = 53.87$

Fig. 12 shows the response factors in function of the given interval of the excitation force based on FEM-Design calculation.

We can say that there are good agreements between the results. However, it should be noted that in Ref. [2] the results of the calculation is given, but the details of the finite element model and calculation method is unclear, therefore there may be differences in the modeling methods.

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/8.3 Footfall analysis of a lightweight floor.str

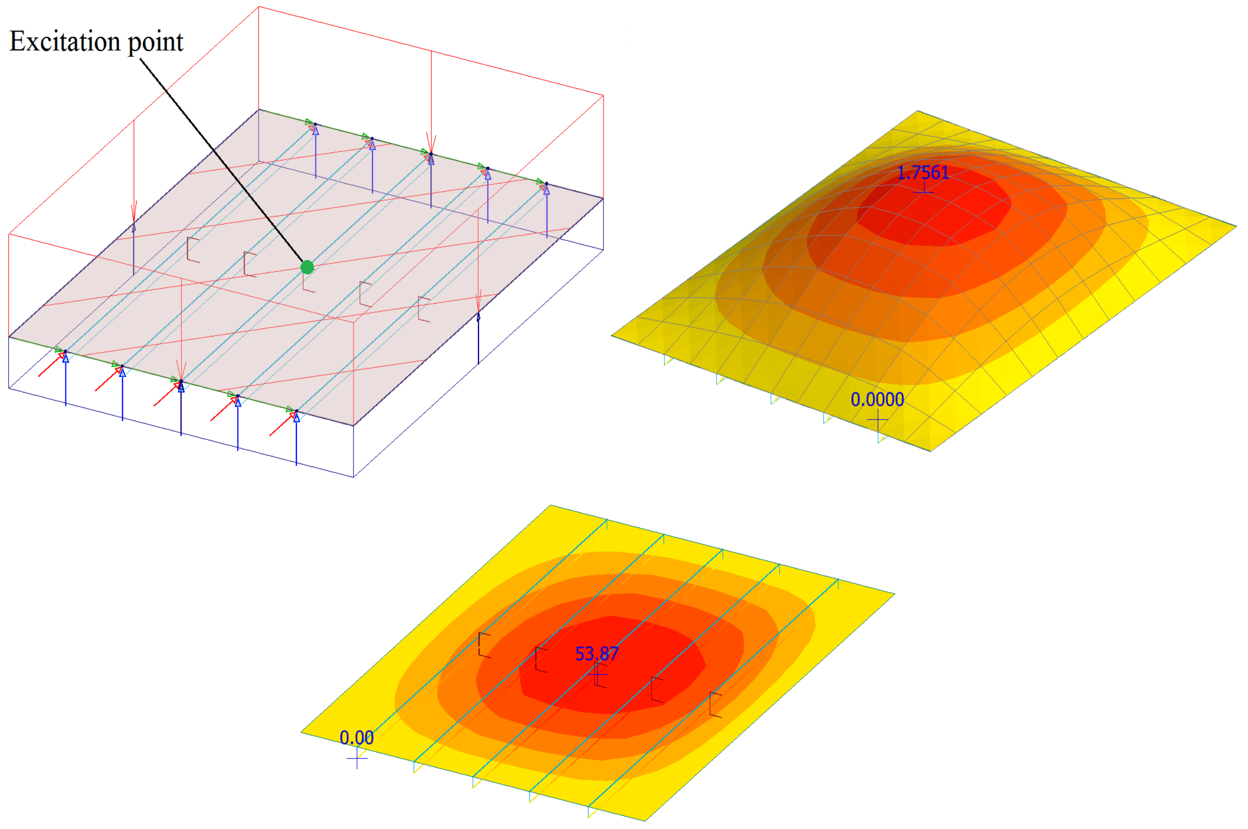


Figure 11 – The model, the first mode shape [-] *and the response factor* [-] *results in FEM-Design*

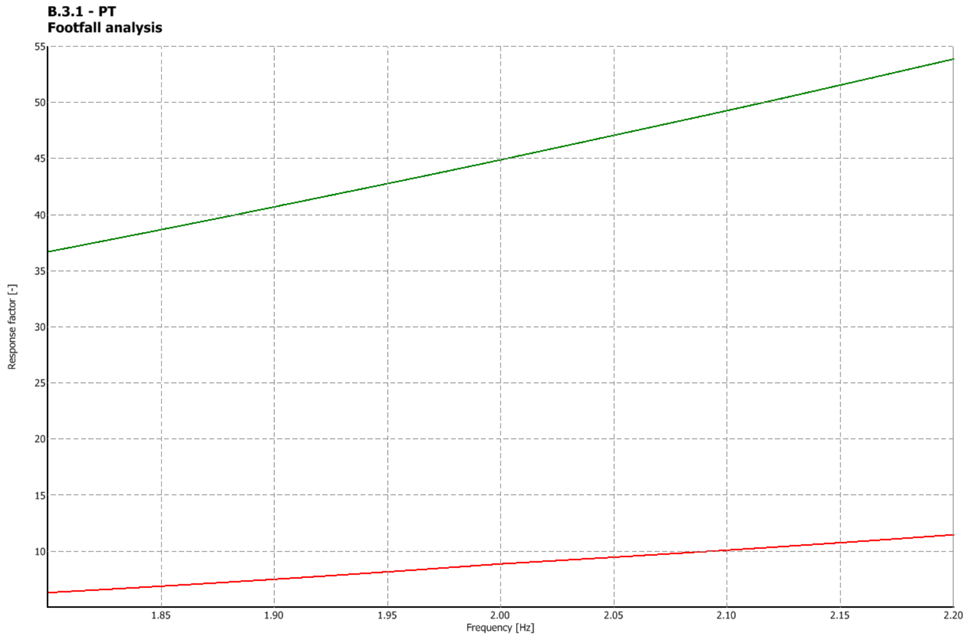


Figure 12 – Response factor in function of excitation frequencies in FEM-Design
Red: steady-state response
Green: transient response

2.4 Footfall analysis of a small stage with rhythmic crowd load

This calculation will be presented according to Danish Annex (Ref. [3]). The floor is a simply supported concrete slab. The half of the slab is a stage where the rhythmic crowd activity will be considered (see Fig. 13).

Inputs for the rhythmic crowd load footfall analysis:

Elastic modulus of concrete	$E = 31$ GPa, $\nu = 0.2$
Thickness of the concrete slab	$t = 250$ mm
Self-weight plus the considered imposed load	$p = 6.75$ kN/m^2
Mean static crowd load (on half of the slab, Fig. 13)	$F_p = 1.0$ kN/m^2
The excitation frequency	$f_p = 3$ Hz
The cut-off eigenfrequency	$f_{cut} = 30$ Hz
Damping	$\zeta = 1.9$ %
Effective number of people	$n_e = 20$
Fourier coefficients (see Subchapter 1.4)	According to Danish Annex Reduced possibility to move about

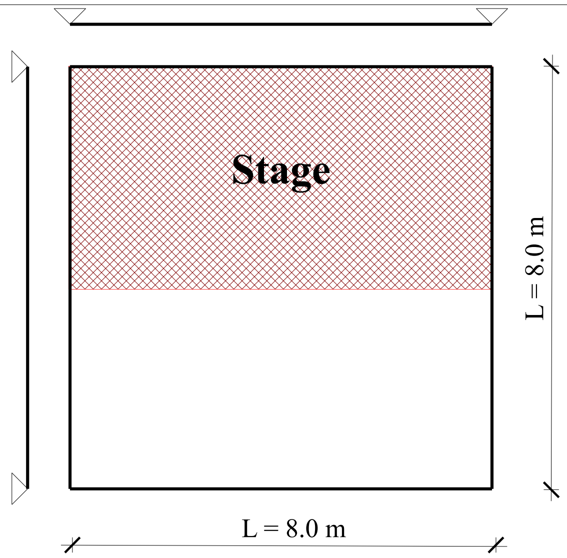


Figure 13 – The slab with the stage

The first eigenfrequency (based on finite element calculation):

$$f_1 = 12\,\mathrm{Hz}$$

In the Danish Annex the logarithmic decrement is given instead of critical damping ratio. The logarithmic decrement with the given critical damping ratio from the inputs:

$$(\delta_s + \delta_p) = 2\pi\,\zeta = 2\pi\,0.019 = 0.12$$

The frequency response factor in the Danish Annex is given with:

$$H_j = \frac{1}{\sqrt{\left(1-\left(\frac{j \cdot f_p}{f_1}\right)^2\right)^2 + \left(\frac{(\delta_s+\delta_p)\, j \cdot f_p}{\pi\, f_1}\right)^2}}$$, therefore:

$$H_1 = \frac{1}{\sqrt{\left(1-\left(\frac{1\cdot 3}{12}\right)^2\right)^2 + \left(\frac{0.12\cdot 1\cdot 3}{\pi\, 12}\right)^2}} = 1.067 \quad ;$$

$$H_2 = \frac{1}{\sqrt{\left(1-\left(\frac{2\cdot 3}{12}\right)^2\right)^2 + \left(\frac{0.12\cdot 2\cdot 3}{\pi\, 12}\right)^2}} = 1.333 \quad ;$$

$$H_3 = \frac{1}{\sqrt{\left(1-\left(\frac{3\cdot 3}{12}\right)^2\right)^2 + \left(\frac{0.12\cdot 3\cdot 3}{\pi\, 12}\right)^2}} = 2.281 \quad .$$

The considered Fourier coefficients including the size reduction factor (see Subchapter 1.4):

$$\alpha_1 K_1 = 0.40 \quad ;$$

$$\alpha_2 K_2 = 0.25\sqrt{0.1+(1-0.1)\frac{1}{20}} = 0.0952 \quad ;$$

$$\alpha_3 K_3 = 0.05\sqrt{0.01+(1-0.01)\frac{1}{20}} = 0.0122 \quad .$$

The dynamic magnification factor for displacements (according to Danish Annex):

$$k_F = \sqrt{\sum_{j=1}^{3}\left(\alpha_j K_j H_j\right)^2} = \sqrt{(0.4\cdot 1.067)^2+(0.0952\cdot 1.333)^2+(0.0122\cdot 2.281)^2}$$

$$k_F = 0.4461$$

The acceleration response factor (according to Danish Annex):

$$k_a = \sqrt{\frac{1}{2}\sum_{j=1}^{3}\left(j^2 \alpha_j K_j H_j\right)^2} = \frac{1}{\sqrt{2}}\sqrt{(1^2\cdot 0.4\cdot 1.067)^2+(2^2\cdot 0.0952\cdot 1.333)^2+(3^2\cdot 0.0122\cdot 2.281)^2}$$

$$k_a = 0.5013$$

The maximum deflection of the slab under the mean static crowd load on the half of the slab (based on a finite element calculation, see Fig. 14):

$$u_p = 0.2132\,\text{mm}$$

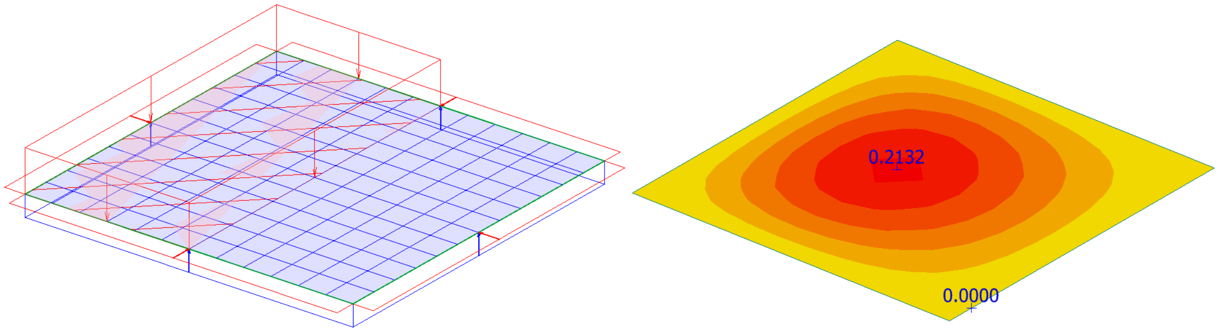


Figure 14 – The slab with the crowd load and the displacements in [mm] *under it in FEM-Design*

The RMS acceleration of the structure induced by the vertical dynamic load (according to Danish Annex):

$$a_a = k_a (2\pi f_p)^2 u_p = 0.5013 \cdot (2\pi 3)^2 0.2132/1000 = 0.03797 \frac{\text{m}}{\text{s}^2}$$

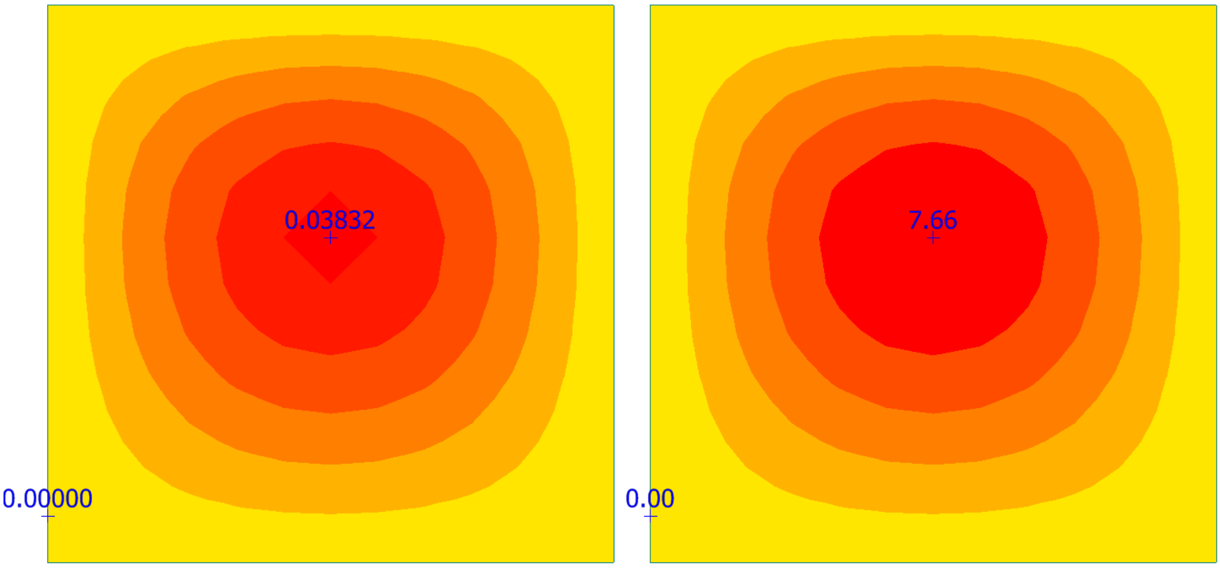
The accelerations and the dynamic magnification factors for displacements based on the FEM-Design calculation (see Fig. 15):

$$a_{FEM} = 0.03832 \frac{\text{m}}{\text{s}^2}$$

$$k_{FEM} = 0.446$$

The difference between the hand calculation and FEM-Design calculation is less than 1%.

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/8.4 Footfall analysis of a small stage with rhythmic crowd load.str



No.	Frequency [Hz]	Dynamic magnification factor [-]
1	12.0	0.446

Figure 15 – The accelerations in [m/s^2] *and the response factors* [-] *in FEM-Design*

References

[1] Chopra A.K., Dynamics of Structures, Prentice Hall,1981.

[2] Smith A. L., Hicks S. J., Devine P. J., Design of Floors for Vibration: A New Approach, The Steel Construction Institute, Ascot, 2009.

[3] DS/EN 1991-1-1 DK NA:2013 Annex C: Rhythmical and syncronised movement of people.

[4] Willford M.R., Young P., A Design Guide for Footfall Induced Vibration of Structures, Concrete Society, 2006.

Notes