FEM-Design

Automatic calculation of flexural buckling length

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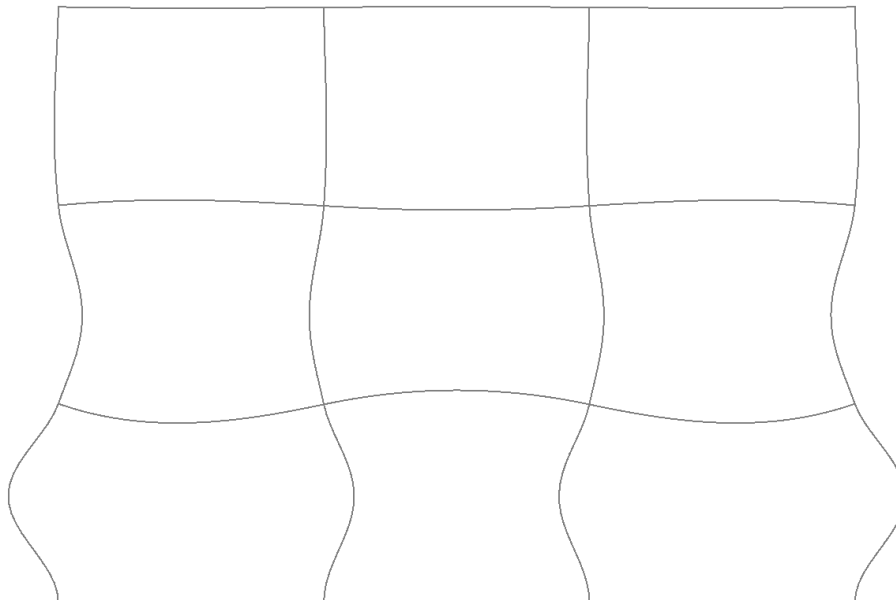
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Contents

List of symbols

Scalars

K_c column stiffness coefficient

K_1 and K_2 stiffness coefficient for the adjacent lengths of the column

K_{ij} effective beam stiffness coefficient

K_T translational stiffness at the far end of the beam, perpendicular to the beam, in one of the principal plain of the cross-section of the beam

K_R rotational stiffness perpendicular to one of the principal plain of the cross-section of the beam

EI_B bending stiffness of the beam in one of the principle axis

EI bending stiffness of the column being design in one of the principle axis

L length of the column being design

L_B length of the beam

β L_{cr} / L buckling length factor

η_1 and η_2 distribution factors at the ends of the column being design

φ rotation at the end of the beam

Abbreviations

$[x' - y' - z']_B$ local system of the beams

$[x' - 1 - 2]_B$ principal system of the beams

$[x' - y' - z']_C$ local system of the column being design

$[x' - 1 - 2]_C$ principal system of the column being design

1 Background information

1.1 Basics

The basic reference for the automatic flexural buckling length calculation is:

ECCS - Rules for Member Stability in EN 1993-1-1: Background documentation and design guidelines.

This documentation presents a general buckling length calculation for regular planar non-sway and sway frames with vertical and horizontal bars. The method is applicable for RC members also and the results of the method is very similar with the buckling length calculation what is indicated in EN 1992-1-1, Chapter 5.8.3.2.

Basically the original (ECCS) method is an approximation therefore there are some limitations to use because a completely general calculation method does not exist. The exact calculation method is the stability eigenvalue calculation based on the specific loads and support conditions of the structure. The method which has been implemented in the program is an approximation with engineering approaches.

The algorithm in the program uses the original method with some reasonable modification to reach a more general solution, according to the further information.

In the following chapters the so-called "column being design" phrase refers to the selected buckling length. The geometric length of a buckling length is the distance between the start point and the end point.

1.2 Theoretical background

The proposal of ECCS - with some reasonable modifications to give a more general method - are as follows.

There are two different theoretical planar models, namely for non-sway and sway frames (see Fig. 1). In the original method the column being design is vertical and all of the connecting beams are horizontal and are in one of the principal plain of the column being design.

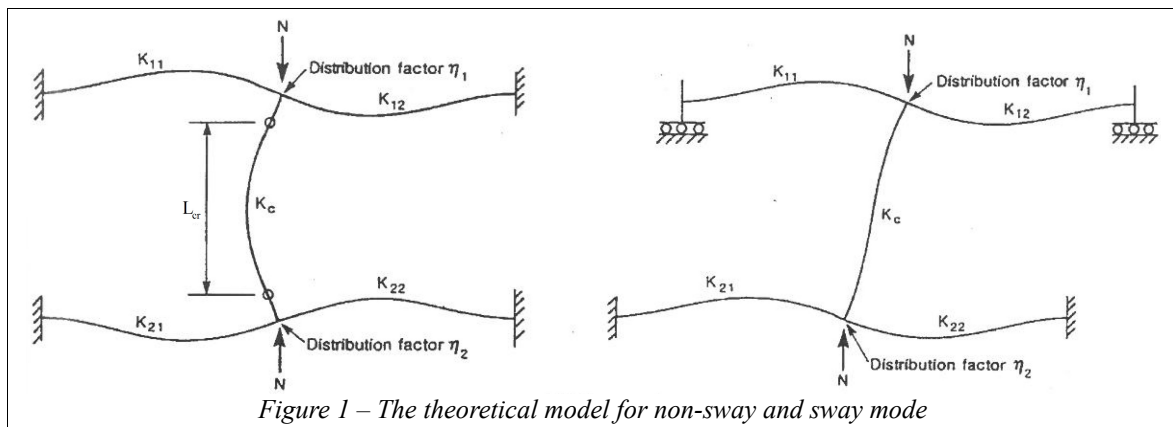In the modified algorithm which has been implemented there are no such limitations.

Figure 1 – The theoretical model for non-sway and sway mode

The following method must be applied in both principle buckling plains of the column being design.

1.2.1 Distribution factors

The distribution factors at the end of the considered column being design (these factors consider the sum of the rotational stiffnesses at the end of the column being design):

$$\eta_1 = \frac{K_c}{K_c + K_{11} + K_{12}} \quad \rightarrow \quad \eta_1 = \frac{K_c}{K_c + \sum_{j=1}^{n} K_{1j}} \quad ,$$

$$\eta_2 = \frac{K_c}{K_c + K_{21} + K_{22}} \quad \rightarrow \quad \eta_2 = \frac{K_c}{K_c + \sum_{j=1}^{n} K_{2j}} \quad ,$$

where the *K* values are the rotational stiffness coefficients. The 0.0 value of this distribution factor means that the end of the column being design is fixed against rotation and the 1.0 value means that the end is hinged.

The K_c column stiffness coefficient by the column being design is:

$$K_c = 4 \frac{EI}{L}$$

The effective stiffness coefficients of the connecting beams at the ends of the column being design in the considered principle buckling plain of the column are K_{ij}.

The i index could be 1 or 2. The 1 means the start point of the column and 2 is the end point of the column respectively (the start and end points depends on the local system of the column). The j index shows the number of the connecting beams at the start or end point.

The calculation method of the effective stiffness coefficients is presented in the following chapters because these values have significant effect on the final automatically calculated buckling length.

If the column being design has a parallel sequel at one end then the distribution factors should be calculated with the following modification:

$$\eta_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} \quad \rightarrow \quad \eta_1 = \frac{K_c + K_1}{K_c + K_1 + \sum_{j=1}^{n} K_{1j}} \quad ,$$

$$\eta_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} \quad \rightarrow \quad \eta_2 = \frac{K_c + K_2}{K_c + K_2 + \sum_{j=1}^{n} K_{2j}} \quad ,$$

where K_1 and K_2 is the effective stiffness coefficient of the continuous (sequel part) column at the start point and at the end point. If there are no other connecting elements at the end of the sequel column part this modification is irrelevant.

All of the presented K values should be interpret in the considered principle plain (buckling length direction) of the column being design.

1.2.2 Effective stiffness coefficients

The effective stiffness coefficients of the connecting beams K_{ij} depends on several things:

   – The far end support condition of the beam (e.g.: fixed, hinged, elastic support, etc.).

   – The connecting elements to the beams at far ends (e.g.: vertical columns, etc.).

   – The previous property depends on whether the column is non-sway or sway.

   – The end release condition of the connecting beam at both ends (e.g.: hinges).

In this chapter the first three things will be discussed and the next subchapter will contain the remaining one.

In general case if the connecting beam has a translational (K_T) and rotational (K_R) point support (spring) at the far end (see Fig. 2) the effective rotational stiffness parameter in the principal plain of the straight beam is the following:

$$K_{ij} = 4 \frac{EI_B \left[K_T L_B^2 \left(K_R L_B + 3\,EI_B \right) + 3\,K_R\,EI_B \right]}{K_T L_B^3 \left(K_R L_B + 4\,EI_B \right) + 12\,EI_B \left(K_R L_B + EI_B \right)} \qquad \text{(Eq. 1)}$$

The K_T value is the translational stiffness at the far end of the beam perpendicular to the axis of the beam in one of the principal plains ($[x' - 1 - 2]_B$, see Fig. 2).

The K_R value is the rotational stiffness at far end of the beam in one of the principal plain (see also Fig. 2).
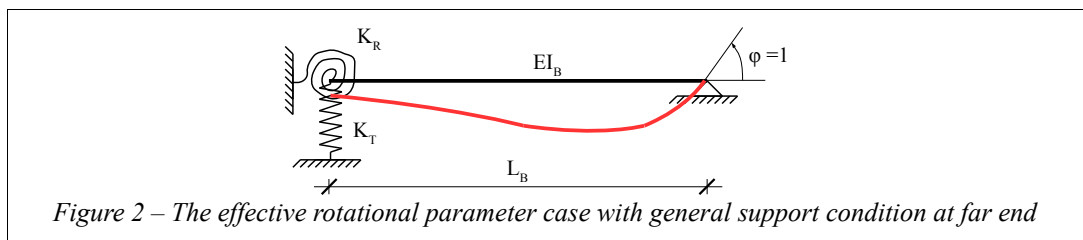

Figure 2 – The effective rotational parameter case with general support condition at far end

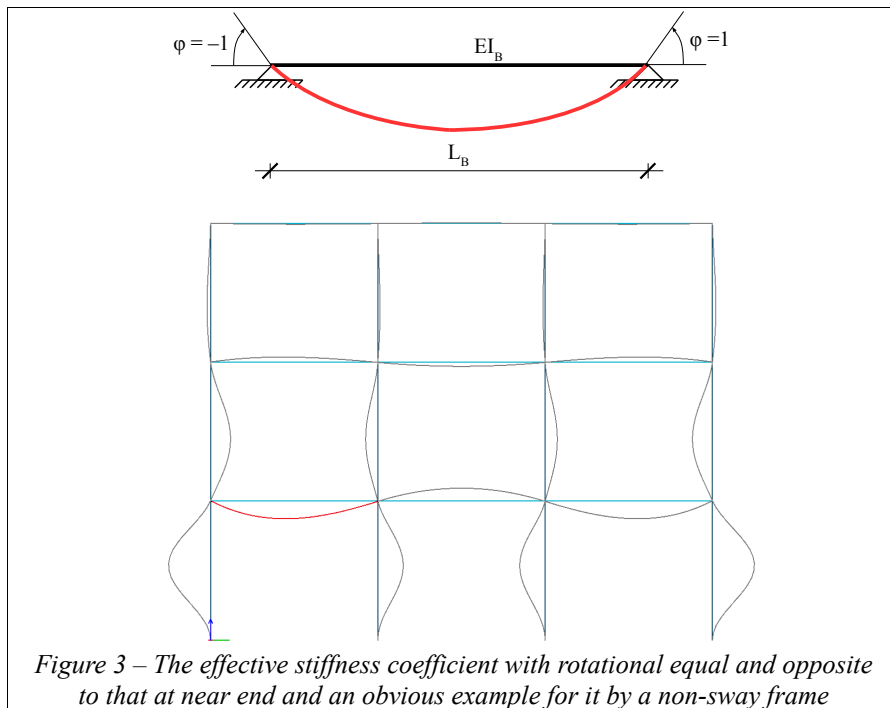Eq. 1 leads to the well-known coefficients in the extreme cases.

For example let's see the following far end support conditions of the connecting beam and calculate the effective stiffness coefficient with Eq. 1:

$K_T \to \infty$ and $K_R \to \infty$ (fixed end)	$K_{ij} = 4\dfrac{EI_B}{L_B}$
$K_T \to \infty$ and $K_R \to 0$ (hinged end)	$K_{ij} = 3\dfrac{EI_B}{L_B}$
$K_T \to 0$ and $K_R \to 0$ (free end)	$K_{ij} = 0$
$K_T \to 0$ and $K_R \to \infty$	$K_{ij} = \dfrac{EI_B}{L_B}$

In a **non-sway frame** if the connecting beam has a connection with an adjacent column (or any other structural part which is not parallel with the local x' axis of the beam and which is not perpendicular to the x' axis of the column being design and which is not a point support) then the rotational stiffness parameter comes from the case when the rotation equal and opposite to that at the near end (single curvature, see Fig. 3). The stiffness parameter (in both principal plains) in this case:

$$K_{ij} = 2\frac{EI_B}{L_B}$$

This is a reasonable approximation to get a correct result in most of the cases (see the first storey columns and the connecting beams at its top end in Fig. 3)

Figure 3 – The effective stiffness coefficient with rotational equal and opposite to that at near end and an obvious example for it by a non-sway frame

In a **sway frame** if the connecting beam has a connection with an adjacent column (or any other structural part which is not parallel with the local x' axis of the beam and which is not perpendicular to the x' axis of the column being design and which is not a point support) then the rotational stiffness parameter comes from the case when the rotation equal to that at the near end (double curvature, see Fig. 4). The stiffness parameter (in both principal plains) in this case:

$$K_{ij} = 6\frac{EI_B}{L_B}$$

This is a reasonable approximation to get a correct result in most cases (see the first storey columns and the connecting beams at its top end in Fig. 4)
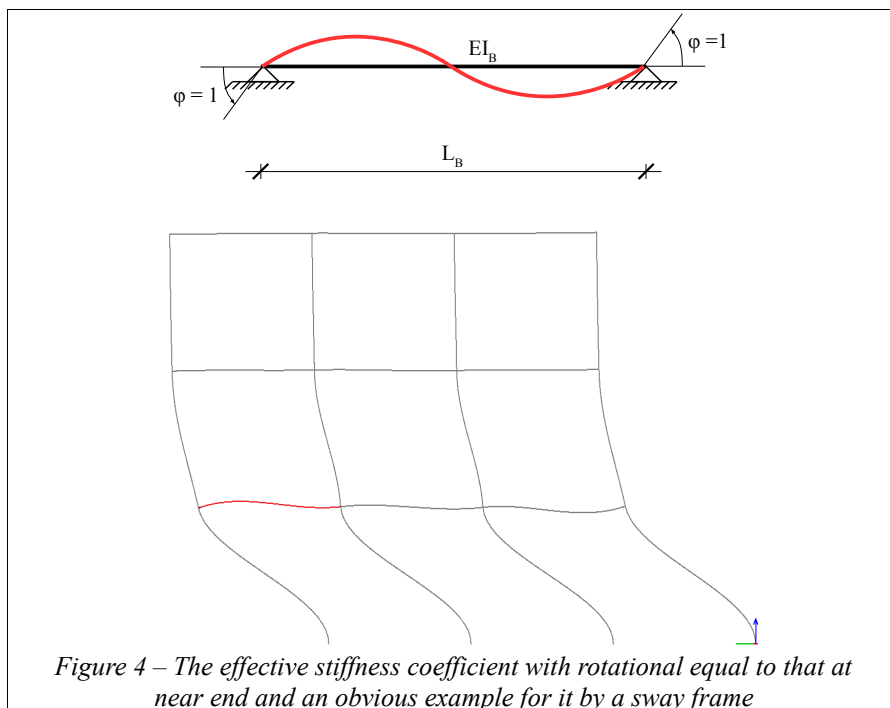

Figure 4 – The effective stiffness coefficient with rotational equal to that at near end and an obvious example for it by a sway frame

1.2.3 Transformation of the point support and effective stiffness coefficients

The connecting beam rotational stiffness parameters if there is a point support at the far end of the beam are as follows:

The point support stiffnesses (3 translational and 3 rotational) should transform from its local system to beam $x' - 1 - 2$ system.

Translational:

$$
\begin{bmatrix} K_{x'} & 0 & 0 \\ 0 & K_{y'} & 0 \\ 0 & 0 & K_{z'} \end{bmatrix}_{ps} \rightarrow \underline{\underline{T}} \begin{bmatrix} K_{x'} & 0 & 0 \\ 0 & K_{y'} & 0 \\ 0 & 0 & K_{z'} \end{bmatrix}_{ps} \underline{\underline{T}}^{\mathrm{T}} = \begin{bmatrix} K_{x'} & & \\ & K_{T1} & \\ & & K_{T2} \end{bmatrix}_{[x'-1-2]B}
$$

Rotational:

$$
\begin{bmatrix} C_{x'} & 0 & 0 \\ 0 & C_{y'} & 0 \\ 0 & 0 & C_{z'} \end{bmatrix}_{ps} \rightarrow \underline{\underline{T}} \begin{bmatrix} C_{x'} & 0 & 0 \\ 0 & C_{y'} & 0 \\ 0 & 0 & C_{z'} \end{bmatrix}_{ps} \underline{\underline{T}}^{\mathrm{T}} = \begin{bmatrix} C_{x'} & & \\ & K_{R1} & \\ & & K_{R2} \end{bmatrix}_{[x'-1-2]B}
$$

The $\underline{\underline{T}}$ transformation matrix here:

$$\underline{\underline{T}} \cdot \underline{u}_{(x'-y'-z')ps} = \underline{u}_{(x'-1'-2')B}$$

With these stiffnesses (K_{T1}, K_{R2} and K_{T2}, K_{R1} in the principal plains) the K_{ij1} and K_{ij2} values should be calculated according to the former general equation (Eq. 1) in the principal directions.

After this step these two rigidity values should transform into the $x' - y' - z'$ local system of the connecting beam.

$$
\begin{bmatrix} K_{ij1} \\ K_{ij2} \end{bmatrix}_B \rightarrow \begin{bmatrix} K_{ijy'} \\ K_{ijz'} \end{bmatrix}_B = \begin{bmatrix} K_{ij1}\cos^2\theta + K_{ij2}\sin^2\theta \\ K_{ij1}\sin^2\theta + K_{ij2}\cos^2\theta \end{bmatrix} \quad ,
$$

where θ is the angle between *1* and y' axis of the beam ($x' - 1 - 2 \rightarrow x' - y' - z'$).

If the settings of the end release at the near end is hinged in y' or z' direction then in that direction the stiffness parameter is set to zero.

As a conservative approximation when the far end is hinged about y' or z' direction then that stiffness parameter is multiplied by 0.75.

By this calculation only the hinged option is considered the other non-zero end release conditions are neglected (see Fig. 2).
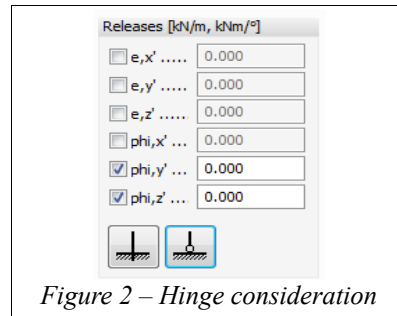
Figure 2 – Hinge consideration

The connecting beam rotational stiffness parameters if there are non-parallel connecting beams, non-parallel columns or shells at the far end of the beam are as follows:

Non-sway case (see the end of the 1.2.2 subchapter):

$$\begin{bmatrix} K_{ij1} \\ K_{ij2} \end{bmatrix}_B = \begin{bmatrix} 2\dfrac{EI_{1B}}{L_B} \\ 2\dfrac{EI_{2B}}{L_B} \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} K_{ijy'} \\ K_{ijz'} \end{bmatrix}_B = \begin{bmatrix} K_{ij1}\cos^2\theta + K_{ij2}\sin^2\theta \\ K_{ij1}\sin^2\theta + K_{ij2}\cos^2\theta \end{bmatrix} \quad ,$$

where θ is the angle between *1* and *y'* axis of the beam ($x' - 1 - 2 \rightarrow x' - y' - z'$).

If the settings of the end release at the near end is hinged in *y'* or *z'* direction then in that direction the stiffness parameter is set to zero.

As a conservative approximation when the far end is hinged about *y'* or *z'* direction then that stiffness parameter is multiplied by 1.5.

Sway case (see the end of the 1.2.2 subchapter):

$$\begin{bmatrix} K_{ijy'} \\ K_{ijz'} \end{bmatrix}_B = \begin{bmatrix} 6\dfrac{EI_{1B}}{L_B} \\ 6\dfrac{EI_{2B}}{L_B} \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} K_{ijy'} \\ K_{ijz'} \end{bmatrix}_B = \begin{bmatrix} K_{ij1}\cos^2\theta + K_{ij2}\sin^2\theta \\ K_{ij1}\sin^2\theta + K_{ij2}\cos^2\theta \end{bmatrix} \quad ,$$

where θ is the angle between *1* and *y'* axis of the beam ($x' - 1 - 2 \rightarrow x' - y' - z'$).

If the settings of the end release at the near end is hinged in *y'* or *z'* direction then in that direction the stiffness parameter is set to zero.

As a conservative approximation when the far end is hinged about *y'* or *z'* direction then that stiffness parameter is multiplied by 0.5.

By these calculations only the hinged option is considered the other non-zero end release conditions are neglected (see Fig. 2).

Calculation of the final effective stiffness coefficients:

After these effective stiffness coefficients are available they should be transformed into the buckling plains (principal plains) of the column being design.

The buckling lengths are relevant in the principal directions of the section. It means that during the distribution factor calculation the end rotational stiffnesses should be transformed into the principal direction of the section (into the principle system of the column being design).

The torsional stiffness of the connecting beams are neglected as a conservative approximation.

$$
\begin{bmatrix} K_{ij1} \\ K_{ij2} \end{bmatrix}_C = \begin{bmatrix} K_{y'_{beam}} T_{22}^2 + K_{z'_{beam}} T_{23}^2 \\ K_{y'_{beam}} T_{32}^2 + K_{z'_{beam}} T_{33}^2 \end{bmatrix} \quad ,
$$

where the $\underline{\underline{T}}$ transformation matrix here:

$\underline{\underline{T}} \cdot \underline{u}_{(x'-y'-z')B} = \underline{u}_{(x'-1-2)C}$

After the summation of the effective rotational stiffnesses at both ends (in $x' - 1 - 2$ column, including the possible point support against rotation at the ends of the column being design), the hinged option of the column being design is considered with reset the relevant values with zero.

By this calculation only the hinged option is considered the other non-zero end release conditions are neglected (see Fig. 2).

1.2.4 Calculation of the buckling lengths

After the calculation of these effective rotational stiffnesses in the principal system of the column being design the buckling length can be determined with the following equations.

The ratio between flexural buckling length and the geometric length of the column being design is (the so-called beta factor):

$$\beta = \frac{L_{cr}}{L}$$

The ECCS value of this ratio with the help of the distribution factors in the two different cases:

Non-sway frame:

$$\beta = \frac{1 + 0.145(\eta_1 + \eta_2) - 0.265\,\eta_1\eta_2}{2 - 0.364(\eta_1 + \eta_2) - 0.247\,\eta_1\eta_2}$$
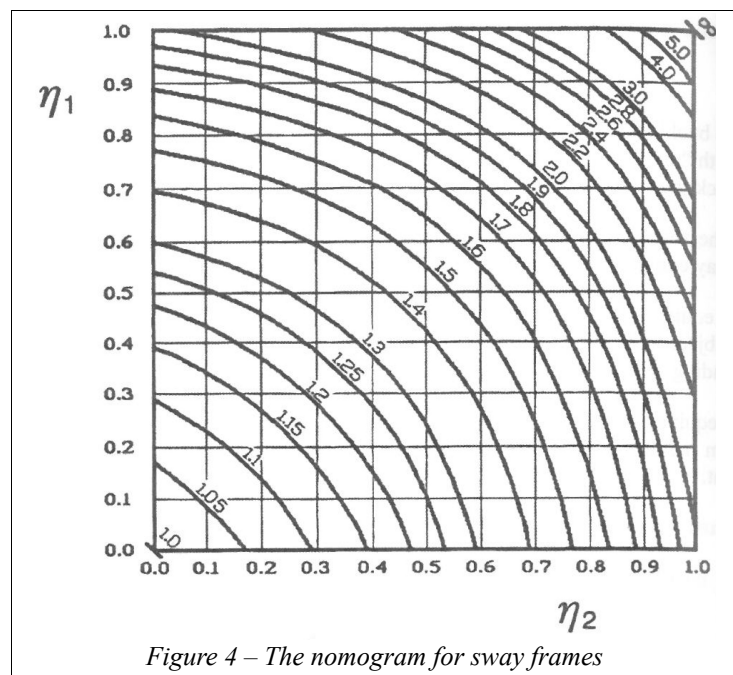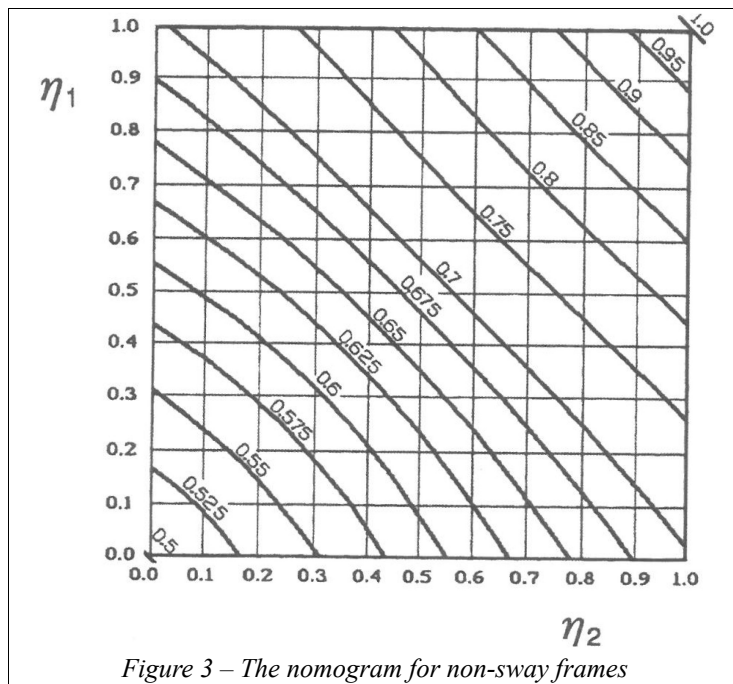
In this case the factor is between 0.5 and 1.0 obviously.

Sway frame:

$$\beta = \sqrt{\frac{1 - 0.2(\eta_1 + \eta_2) - 0.12\,\eta_1\eta_2}{1 - 0.8(\eta_1 + \eta_2) + 0.6\,\eta_1\eta_2}}$$

In this case the factor is between 1.0 and +∞. A reasonable upper limit is set to 10.0 in FEM-Design.

The nomogram representation of these two cases are shown in Fig. 3 and 4.

Figure 3 – The nomogram for non-sway frames

Figure 4 – The nomogram for sway frames

1.2.5 Restrictions and other comments

Consideration of slabs:

As a rough approximation in case of non-sway and sway case if <u>only a plate or a wall</u> is available (without any other structural elements) at the end of the column being design the distribution factor will be 0.0 assumed as a completely fixed end of the column. This is a rough estimation.

Consideration of trusses.

The calculation of the effective stiffness coefficient is relevant for connecting beams and columns as well, but the effective stiffness of a truss element is zero according to the hinged-hinged ends.

Consideration of variable cross-sections.

If the cross-section of the column being design or the connecting beam is variable along its axis then the program compare that at the start point or at the end point cross-section has a smaller polar moment of inertia. The considered second moment of inertia (which is necessary by the bending stiffness calculation) will come from that cross-section where the mentioned polar moment of inertia is smaller (at the start or at the end point of the bar).

Consideration of composite bars.

By the composite bars the considered bending stiffness will be the bending stiffness which belongs to the homogenized cross-section based on the stability elastic modulus of the concrete.

Consideration of bilinear point supports.

By the bilinear point supports always the greater initial stiffness will be considered.

2 Verification examples

2.1 Concrete frame building

In this example we will calculate the buckling lengths of the indicated isolated columns (C.1; C.4, see Fig. 5) according to EN 1992-1-1:2004 Chapter 5.8.3.2. After the hand calculation we will compare the results with the FEM-Design automatic buckling length calculation results.

The geometry is shown in Fig. 5. The material is C25/30 the columns have 300/300 mm, the beams have 300/500 mm cross-sections. We will calculate the buckling lengths of the middle isolated columns at the ground floor and at the first floor. The supports are fixed at the bottom of the ground floor columns.
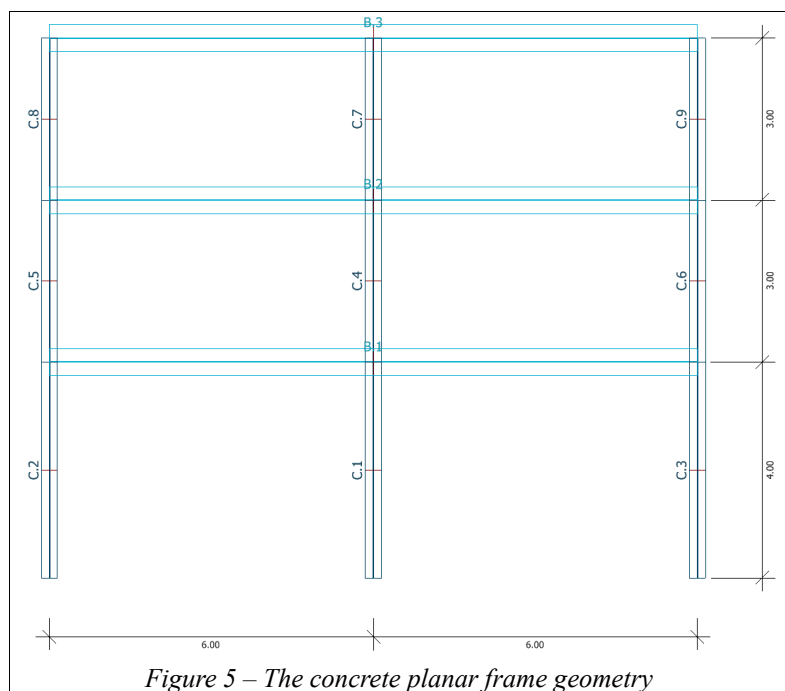

Figure 5 – The concrete planar frame geometry

2.1.1 Non-sway case

If the frame is a non-sway frame the method according to EN 1992-1-1:2004 Chapter 5.8.3.2. is the following:

The bending stiffness of the columns:

$$EI_c = 31000000 \cdot \frac{0.3^4}{12} = 20925 \, \text{kNm}^2$$

The bending stiffness of the beams in the relevant direction:

$$EI_c = 31000000 \cdot \frac{0.3 \cdot 0.5^3}{12} = 96875 \, \text{kNm}^2$$

C.1 column (see Fig. 5):

The distribution factors:

At bottom:

$k_1 = 0$ (fixed support);

At top:

$$k_2 = \frac{(EI_c/L_{c4})_{\text{above}} + (EI_c/L_{c1})_{\text{below}}}{\sum c\, EI_b/L_b} = \frac{\dfrac{20925}{3} + \dfrac{20925}{4}}{2\dfrac{96875}{6} + 2\dfrac{96875}{6}} = 0.189$$

By the beams rotational stiffnesses we assumed a single curvature due to the non-sway situation.

The beta factor of the buckling length:

$$\beta_1 = \frac{L_{cr1}}{L_{c1}} = 0.5 \cdot \sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right)\left(1 + \frac{k_2}{0.45 + k_2}\right)} = 0.5 \cdot \sqrt{\left(1 + \frac{0}{0.45 + 0}\right)\left(1 + \frac{0.1896}{0.45 + 0.189}\right)} = 0.569$$

C.4 column (see Fig. 5):

The distribution factors:

At bottom:

$$k_1 = \frac{(EI_c/L_{c4})_{\text{above}} + (EI_c/L_{c1})_{\text{below}}}{\sum c\, EI_b/L_b} = \frac{\dfrac{20925}{3} + \dfrac{20925}{4}}{2\dfrac{96875}{6} + 2\dfrac{96875}{6}} = 0.189$$

By the beams rotational stiffnesses we assumed a single curvature due to the non-sway situation.

At top:

$$k_2 = \frac{(EI_c/L_{c7})_{\text{above}} + (EI_c/L_{c4})_{\text{below}}}{\sum c\, EI_b/L_b} = \frac{\dfrac{20925}{3} + \dfrac{20925}{3}}{2\dfrac{96875}{6} + 2\dfrac{96875}{6}} = 0.216$$

By the beams rotational stiffnesses we assumed a single curvature due to the non-sway situation.
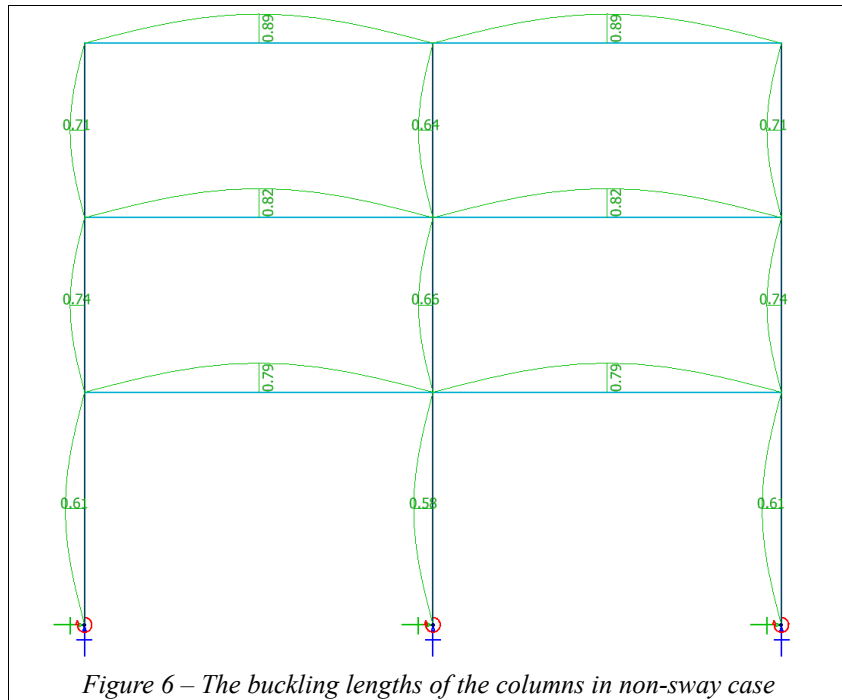
The beta factor of the buckling length:

$$\beta_4 = \frac{L_{cr4}}{L_{c4}} = 0.5 \cdot \sqrt{\left(1 + \frac{0.189}{0.45 + 0.189}\right)\left(1 + \frac{0.216}{0.45 + 0.216}\right)} = 0.655$$

Based on FEM-Design auto buckling length calculation method the results are:

$$\beta_{1FEM} = 0.576$$

$$\beta_{4FEM} = 0.663$$

The difference between the calculations is less than 1.5%. Fig. 6 shows the results based on FEM-Design.

Figure 6 – The buckling lengths of the columns in non-sway case

2.1.2 Sway case

If the frame is a sway frame the method according to EN 1992-1-1:2004 Chapter 5.8.3.2. is the following:

C.1 column (see Fig. 5):

The distribution factors:

At bottom:

$k_1 = 0$ (fixed support);

At top:

$$k_2 = \frac{(EI_c / L_{c4})_{above} + (EI_c / L_{c1})_{below}}{\sum c \, EI_b / L_b} = \frac{\dfrac{20925}{3} + \dfrac{20925}{4}}{6\dfrac{96875}{6} + 6\dfrac{96875}{6}} = 0.063$$

By the beams rotational stiffnesses we assumed double curvature due to the sway situation.

The beta factor of the buckling length:

$$\beta_1 = \frac{L_{cr1}}{L_{c1}} = \max\left[\begin{array}{c} \sqrt{1 + 10\dfrac{k_1 \cdot k_2}{k_1 + k_2}} \\ \left(1 + \dfrac{k_1}{1 + k_1}\right)\cdot\left(1 + \dfrac{k_2}{1 + k_2}\right) \end{array} \right] = \max\left[\begin{array}{c} \sqrt{1 + 10\dfrac{0 \cdot 0.063}{0 + 0.063}} \\ \left(1 + \dfrac{0}{1 + 0}\right)\cdot\left(1 + \dfrac{0.063}{1 + 0.063}\right) \end{array} \right] = 1.06$$

C.4 column (see Fig. 5):

The distribution factors:

At bottom:

$$k_1 = \frac{(EI_c / L_{c4})_{above} + (EI_c / L_{c1})_{below}}{\sum c \, EI_b / L_b} = \frac{\dfrac{20925}{3} + \dfrac{20925}{4}}{6\dfrac{96875}{6} + 6\dfrac{96875}{6}} = 0.063$$

By the beams rotational stiffnesses we assumed double curvature due to the sway situation.

At top:

$$k_2 = \frac{(EI_c / L_{c7})_{above} + (EI_c / L_{c4})_{below}}{\sum c \, EI_b / L_b} = \frac{\dfrac{20925}{3} + \dfrac{20925}{3}}{6\dfrac{96875}{6} + 6\dfrac{96875}{6}} = 0.072$$

By the beams rotational stiffnesses we assumed double curvature due to the sway situation.
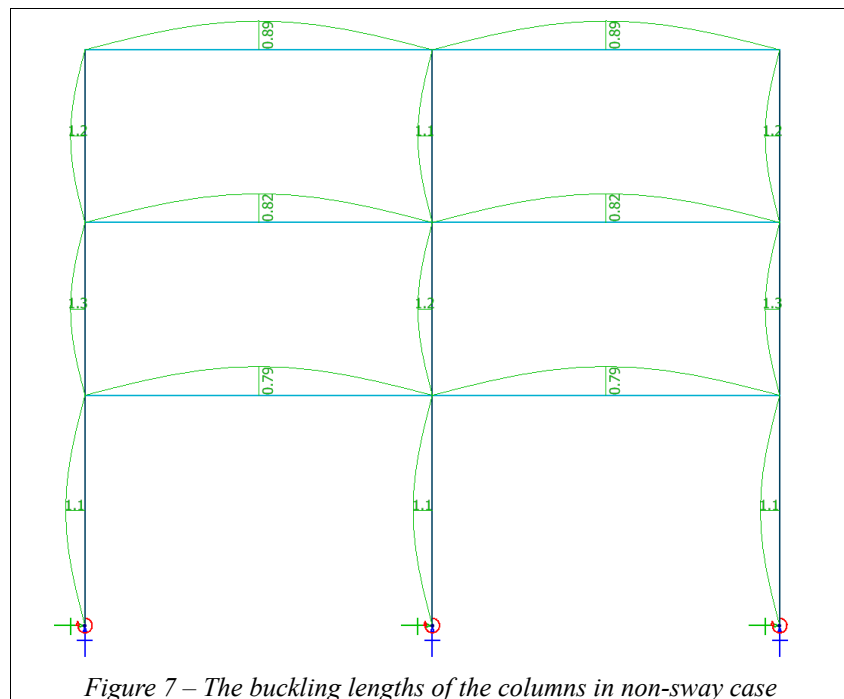
The beta factor of the buckling length:

$$\beta_4 = \frac{L_{cr4}}{L_4} = \max\left[\begin{matrix} \sqrt{1+10\dfrac{0.063\cdot 0.072}{0.063+0.072}} \\ \left(1+\dfrac{0.063}{1+0.063}\right)\cdot\left(1+\dfrac{0.072}{1+0.072}\right) \end{matrix}\right] = 1.156$$

Based on FEM-Design auto buckling length calculation method the results are:

$\beta_{1FEM} = 1.07$

$\beta_{4FEM} = 1.15$

The difference between the calculations is less than 1%. Fig. 7 shows the results based on FEM-Design.

Figure 7 – The buckling lengths of the columns in non-sway case

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/9.5.1 Auto Buckling length concrete building.str

2.2 Steel frame building

In this example we will calculate the buckling lengths of the indicated isolated columns (C.2; C.6, see Fig. 8) according to the method in Ref. [1] which is basically indentical with the given method in the former ENV 1993-1-1:1992 Annex E. After the hand calculation we will compare the results with FEM-Design automatic buckling length calculation results.

The geometry is shown in Fig. 8. The material is S235, the outer columns have HEB220, the inner columns have HEB260, the beams have IPE450 and the beams at the roof have IPE360 cross-sections. We will calculate the buckling lengths of the middle isolated columns at the ground floor and at the first floor. <u>The supports are hinged at the bottom of the ground floor columns.</u>
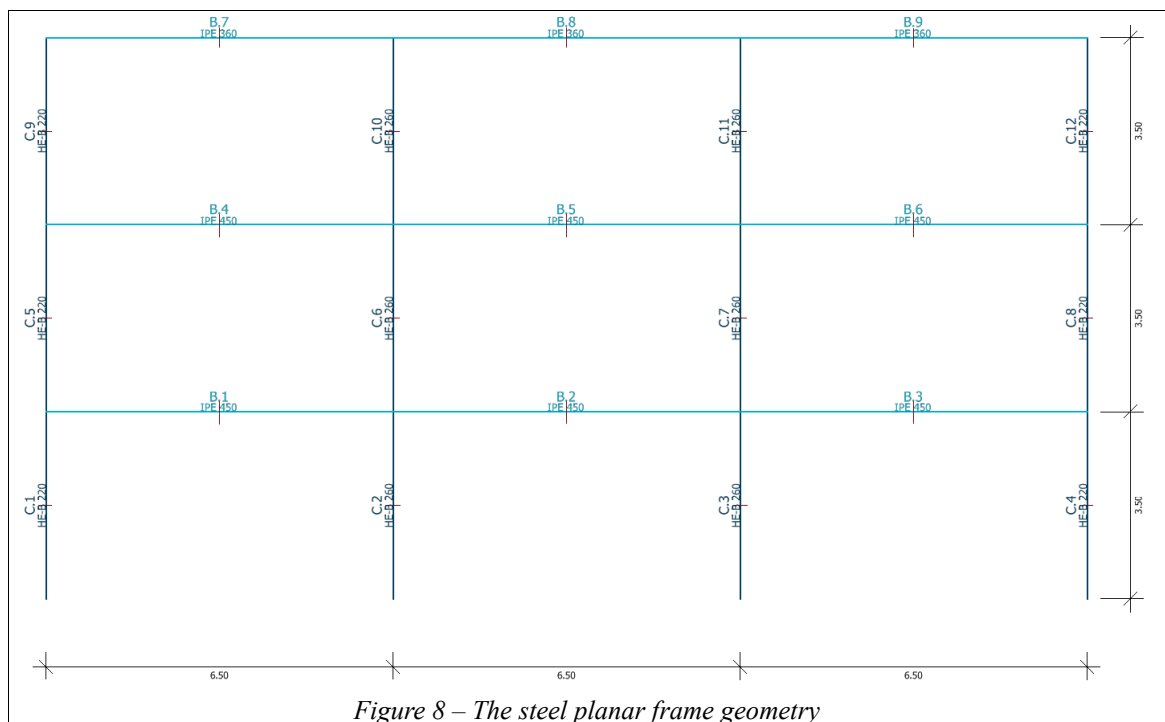
2.2.1 Non-sway case

If the frame is a non-sway frame the method according to Ref. [1] is the following:

The bending stiffness of the columns (HEB260):

$$EI_c = 210000000 \cdot 0.0001492 = 31332\,\text{kNm}^2$$

The bending stiffness of the beams (IPE450):

$$EI_b = 210000000 \cdot 0.0003374 = 70854\,\text{kNm}^2$$

Figure 8 – The steel planar frame geometry

The rotational stiffness coefficient of the columns being analyzed (C.2 and C.6):

$$K_c = 4\,\frac{EI_c}{L_c} = 4\,\frac{31332}{3.5} = 35808\,\text{kNm}$$

C.2 column (see Fig. 8):

The distribution factors:

At bottom:

$$\eta_1 = \frac{K_c}{K_c} = 1.0 \quad \text{(hinged support)}$$

At top:

$$\eta_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} = \frac{4\dfrac{EI_c}{L_c} + 4\dfrac{EI_c}{L_c}}{4\dfrac{EI_c}{L_c} + 4\dfrac{EI_c}{L_c} + 2\dfrac{EI_b}{L_b} + 2\dfrac{EI_b}{L_b}} = \frac{35808 + 35808}{35808 + 35808 + 2\dfrac{70854}{6.5} + 2\dfrac{70854}{6.5}}$$

$$\eta_2 = 0.622$$

By the beams rotational stiffnesses we assumed a single curvature due to the non-sway situation.

The beta factor of the buckling length:

$$\beta_2 = \frac{1 + 0.145(\eta_1 + \eta_2) - 0.265\,\eta_1\eta_2}{2 - 0.364(\eta_1 + \eta_2) - 0.247\eta_1\eta_2} = \frac{1 + 0.145(1.0 + 0.622) - 0.265 \cdot 1.0 \cdot 0.622}{2 - 0.364(1.0 + 0.622) - 0.247 \cdot 1.0 \cdot 0.622}$$

$$\beta_2 = 0.852$$

C.6 column (see Fig. 8):

The distribution factors:

At bottom:

$$\eta_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} = \frac{4\dfrac{EI_c}{L_c} + 4\dfrac{EI_c}{L_c}}{4\dfrac{EI_c}{L_c} + 4\dfrac{EI_c}{L_c} + 2\dfrac{EI_b}{L_b} + 2\dfrac{EI_b}{L_b}} = \frac{35808 + 35808}{35808 + 35808 + 2\dfrac{70854}{6.5} + 2\dfrac{70854}{6.5}}$$

$$\eta_1 = 0.622$$

By the beams rotational stiffnesses we assumed a single curvature due to the non-sway situation.

At top:

$$\eta_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} = \frac{4\dfrac{EI_c}{L_c} + 4\dfrac{EI_c}{L_c}}{4\dfrac{EI_c}{L_c} + 4\dfrac{EI_c}{L_c} + 2\dfrac{EI_b}{L_b} + 2\dfrac{EI_b}{L_b}} = \frac{35808 + 35808}{35808 + 35808 + 2\dfrac{70854}{6.5} + 2\dfrac{70854}{6.5}}$$

$$\eta_2 = 0.622$$

By the beams rotational stiffnesses we assumed a single curvature due to the non-sway situation.

The beta factor of the buckling length:

$$\beta_6 = \frac{1+0.145(\eta_1+\eta_2)-0.265\,\eta_1\eta_2}{2-0.364(\eta_1+\eta_2)-0.247\,\eta_1\eta_2} = \frac{1+0.145(0.622+0.622)-0.265\cdot0.622\cdot0.622}{2-0.364(0.622+0.622)-0.247\cdot0.622\cdot0.622}$$

$\beta_6 = 0.743$

Based on FEM-Design auto buckling length calculation method the results are:

$\beta_{2FEM} = 0.852$

$\beta_{6FEM} = 0.742$
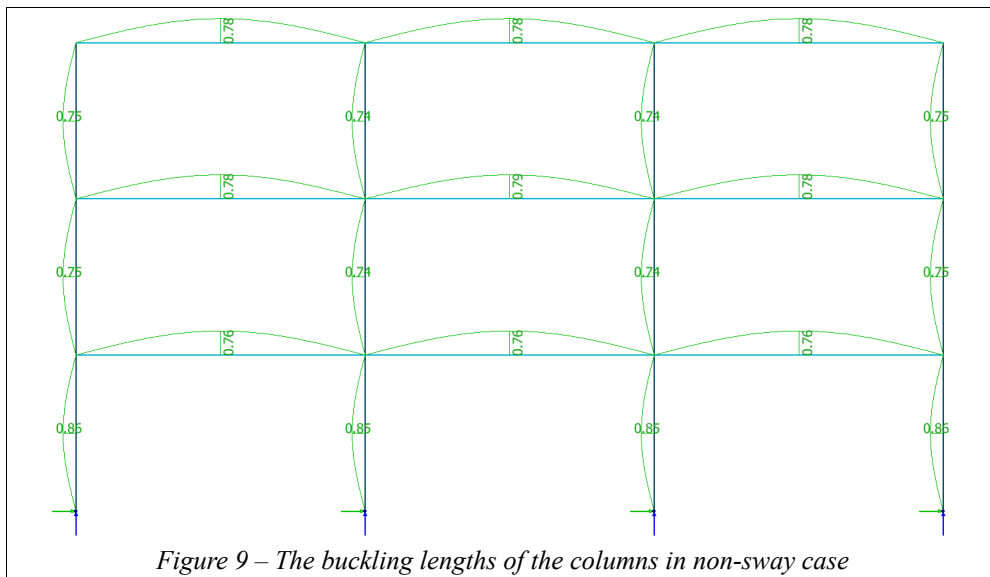
The calculations are identical to each other (see Fig. 9).

Figure 9 – The buckling lengths of the columns in non-sway case

2.2.2 Sway case

If the frame is a sway frame the method according to Ref. [1] is the following:

C.2 column (see Fig. 8):

The distribution factors:

At bottom:

$$\eta_1 = \frac{K_c}{K_c} = 1.0 \quad \text{(hinged support)}$$

At top:

$$\eta_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} = \frac{4\frac{EI_c}{L_c} + 4\frac{EI_c}{L_c}}{4\frac{EI_c}{L_c} + 4\frac{EI_c}{L_c} + 6\frac{EI_b}{L_b} + 6\frac{EI_b}{L_b}} = \frac{35808 + 35808}{35808 + 35808 + 6\frac{70854}{6.5} + 6\frac{70854}{6.5}}$$

$$\eta_2 = 0.354$$

By the beams rotational stiffnesses we assumed double curvature due to the sway situation.

The beta factor of the buckling length:

$$\beta_2 = \sqrt{\frac{1 - 0.2(1.0 + 0.354) - 0.12 \cdot 1.0 \cdot 0.354}{1 - 0.8(1.0 + 0.354) + 0.6 \cdot 1.0 \cdot 0.354}}$$

$$\beta_2 = 2.305$$

C.6 column (see Fig. 8):

The distribution factors:

At bottom:

$$\eta_1 = \frac{K_c + K_1}{K_c + K_1 + K_{11} + K_{12}} = \frac{4\frac{EI_c}{L_c} + 4\frac{EI_c}{L_c}}{4\frac{EI_c}{L_c} + 4\frac{EI_c}{L_c} + 6\frac{EI_b}{L_b} + 6\frac{EI_b}{L_b}} = \frac{35808 + 35808}{35808 + 35808 + 6\frac{70854}{6.5} + 6\frac{70854}{6.5}}$$

$$\eta_1 = 0.354$$

By the beams rotational stiffnesses we assumed double curvature due to the sway situation.

At top:

$$\eta_2 = \frac{K_c + K_2}{K_c + K_2 + K_{21} + K_{22}} = \frac{4\frac{EI_c}{L_c} + 4\frac{EI_c}{L_c}}{4\frac{EI_c}{L_c} + 4\frac{EI_c}{L_c} + 6\frac{EI_b}{L_b} + 6\frac{EI_b}{L_b}} = \frac{35808 + 35808}{35808 + 35808 + 6\frac{70854}{6.5} + 6\frac{70854}{6.5}}$$

$$\eta_2 = 0.354$$

By the beams rotational stiffnesses we assumed double curvature due to the sway situation.

The beta factor of the buckling length:

$$\beta_6 = \sqrt{\frac{1 - 0.2(0.354 + 0.354) - 0.12 \cdot 0.354 \cdot 0.354}{1 - 0.8(0.354 + 0.354) + 0.6 \cdot 0.354 \cdot 0.354}}$$
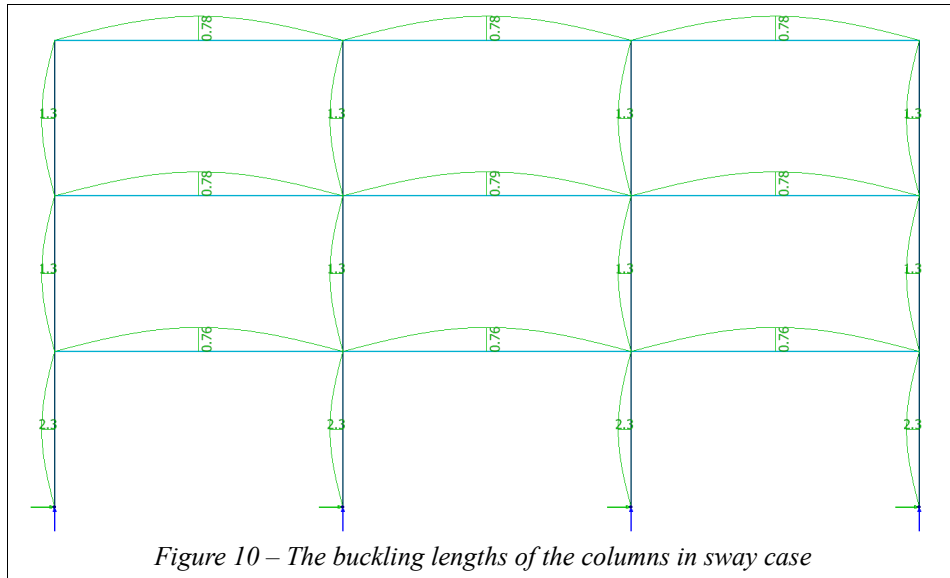
$$\beta_6 = 1.287$$

Based on FEM-Design auto buckling length calculation method the results are:

$\beta_{2FEM} = 2.31$

$\beta_{6FEM} = 1.29$

The calculations are identical to each other (see Fig. 10).

Figure 10 – The buckling lengths of the columns in sway case

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/9.5.2 Auto Buckling length steel building.str

2.3 A column and a supporting beam with various angles

Fig. 11 shows the analyzed problem. The vertical column is HEB260 and the bottom is fixed. The horizontal supporting beam is IPE360 and connected to the upper end of the column (the other end of the beam is simply supported). The angle of the connecting beam is varied between 0º-90º. The plain of the various angle beams is perpendicular to the column (see Fig. 11) thus the supporting beam is always horizontal.

The connecting beam rigidity has effect on the stiff and the weak buckling lengths of the column. After the calculation of the buckling lengths of the column based on the solution of the stability eigenvalue problem (stability calculation) we compared the beta factors with the FEM-Design automatic flexural buckling calculation results.

Figure 11 – The fixed column with various angle supporting beam
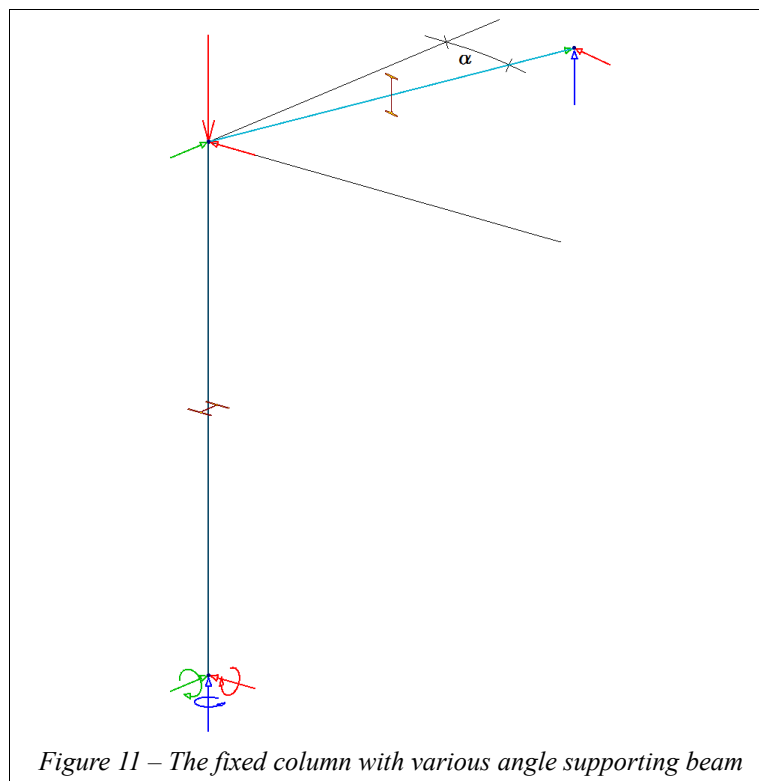
Around the stiff direction the buckling length is increasing because the supporting effect of the connecting beam is decreasing. Around the weak direction the buckling length is decreasing because the supporting effect of the connecting beam is increasing (see Fig. 11, Table 1 and 2).

The critical forces of the hinged-hinged column (so-called Euler force) based on the stability calculation in FEM-Design are:

$F_{cr1} = 11672\,\text{kN}$ around stiff direction,

$F_{cr2} = 4230\,\text{kN}$ around weak direction.

Be careful, these values contain the shear deformation and not only the deformation from bending because in FEM-Design the beam modell is the Timoshenko modell.

For example the beta factor around stiff direction based on the solution of the stability eigenvalue problem when the α angle is equal to 76°:

$$\beta_{76°}^{stiff} = \sqrt{\frac{F_{cr1}}{F_{76°}^{stiff}}} = \sqrt{\frac{11672}{22877}} = 0.714$$

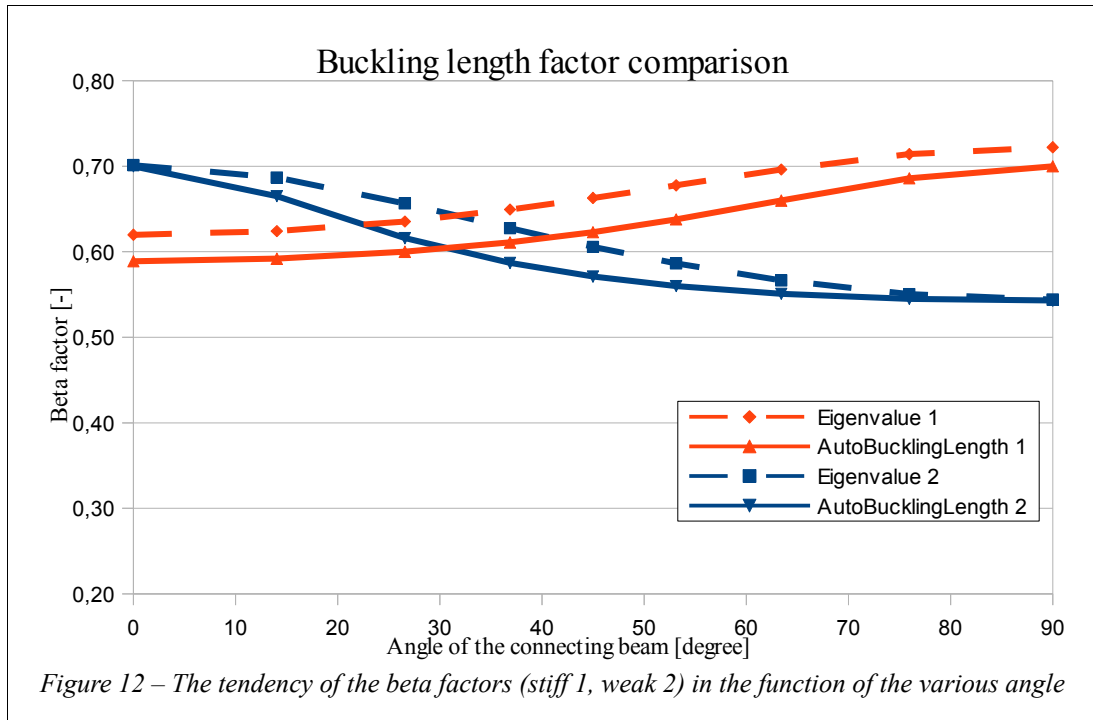| Angle	Critical load	Beta factor	Beta factor	Difference
[degree]	[kN]	Eigenvalue	AutoBucklingLength	[-]
0	30373	0,620	0,589	-0,0499
14	29948	0,624	0,592	-0,0517
27	28899	0,636	0,600	-0,0559
37	27665	0,650	0,611	-0,0593
45	26552	0,663	0,623	-0,0604
53	25410	0,678	0,638	-0,0587
63	24076	0,696	0,660	-0,0521
76	22877	0,714	0,686	-0,0396
90	22372	0,722	0,700	-0,0309

Table 1 – The beta factor around the stiff direction in the function of the given angle

Angle	Critical load	Beta factor	Beta factor	Difference
[degree]	[kN]	Eigenvalue	AutoBucklingLength	[-]
0	8600	0,701	0,700	-0,0019
14	8963	0,687	0,665	-0,0320
27	9811	0,657	0,616	-0,0619
37	10737	0,628	0,587	-0,0648
45	11523	0,606	0,571	-0,0576
53	12295	0,587	0,560	-0,0453
63	13171	0,567	0,551	-0,0277
76	13952	0,551	0,545	-0,0102
90	14284	0,544	0,543	-0,0022

Table 2 – The beta factor around the weak direction in the function of the given angle

The differences between the two calculation methods are less than 6% (see Table 1 and 2). In FEM-Design by the automatic beta factor calculation the column was assumed as a non-sway column according to the original supporting condition (see Fig. 11).

Fig. 12 shows the tendency of the beta factors in function of the supporting beam angle.

Figure 12 – The tendency of the beta factors (stiff 1, weak 2) in the function of the various angle

Download link to the example file:
http://download.strusoft.com/FEM-Design/inst180x/models/9.5.3 A column and a supporting beam with various angles.str

References

[1] ECCS Technical Committee 8 – Stability, Rules for Member Stability in EN 1993-1-1: Background documentation and design guidelines, ECCS, 2006.

Notes